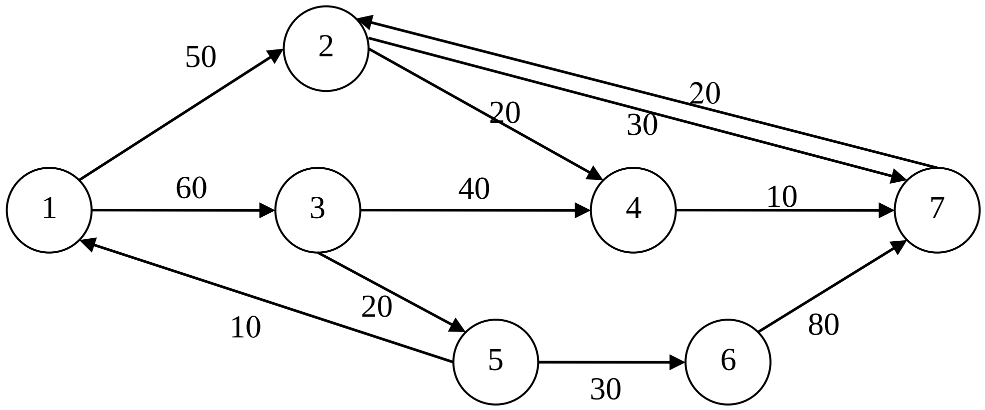
Question 1 (60%)

Given a directed graph, where the values indicate upper bounds:



a. (15%) List all cuts between node 1 and node 7. Find the minimum cut (or cuts).
b. (15%) Find the maximum flow between 1 and 7 using the Augmenting Algorithm. List all algorithmic steps.
c. (15%) Find the maximum flow between 1 and 7 using the Simplex Algorithm. Show the objective function and constraints. List the algorithmic steps. You can use any available software (Solver, Lindo, etc.)
d. (10%) Formulate the dual problem. Show the objective function and constraints.
e. (5%) find the range of the capacity of arc (3, 4) that does NOT change the maximum flow.

Question 2 (40%)

The following table lists the arcs of a directed graph:

Arc capacity	Cost of Flow[1]	To Node	From Node
40	2	2	1
90	3	4	1
90	3	3	2
20	1	5	2
40	2	6	3
40	2	5	4
160	4	7	4
20	1	6	5
40	2	8	5
40	2	9	6
90	3	8	7
40	2	9	8

a. (10%) List all simple paths between 1 and 9 and calculate each path cost.
b. (10%) Find the maximum flow between 1 and 9 and the flow distribution in the network.
c. (10%) In which arc (or arcs) you should increase the capacity, in order to increase the maximum flow? Take into account the cost of each arc.
d. (10%) Is it possible to decrease the total cost in the solution found in item (b) without decreasing the maximum flow?

[1] Each use of a specific arc will incur the mentioned cost, regardless of the volume of flow.