**Question 1 (60%)**

Given a directed graph, where the values indicate upper bounds:

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1. (15%) List all cuts between node 1 and node 7. Find the minimum cut (or cuts).
2. (15%) Find the maximum flow between 1 and 7 using the Augmenting Algorithm. List all algorithmic steps.
3. (15%) Find the maximum flow between 1 and 7 using the Simplex Algorithm. Show the objective function and constraints. List the algorithmic steps. You can use any available software (Solver, Lindo, etc.)
4. (10%) Formulate the dual problem. Show the objective function and constraints.
5. (5%) find the range of the capacity of arc (3, 4) that does NOT change the maximum flow.

**Question 2 (40%)**

The following table lists the arcs of a directed graph:

|  |  |  |  |
| --- | --- | --- | --- |
| From Node | To Node | Cost of Flow[[1]](#footnote-1) | Arc capacity |
| 1 | 2 | 2 | 40 |
| 1 | 4 | 3 | 90 |
| 2 | 3 | 3 | 90 |
| 2 | 5 | 1 | 20 |
| 3 | 6 | 2 | 40 |
| 4 | 5 | 2 | 40 |
| 4 | 7 | 4 | 160 |
| 5 | 6 | 1 | 20 |
| 5 | 8 | 2 | 40 |
| 6 | 9 | 2 | 40 |
| 7 | 8 | 3 | 90 |
| 8 | 9 | 2 | 40 |

1. (10%) List all simple paths between 1 and 9 and calculate each path cost.
2. (10%) Find the maximum flow between 1 and 9 and the flow distribution in the network.
3. (10%) In which arc (or arcs) you should increase the capacity, in order to increase the maximum flow? Take into account the cost of each arc.
4. (10%) Is it possible to decrease the total cost in the solution found in item (b) without decreasing the maximum flow?

1. Each use of a specific arc will incur the mentioned cost, regardless of the volume of flow. [↑](#footnote-ref-1)