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Robust Total Least Squares Based Optic Flow Computation

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Abstract This paper considers the problem of finding a robust solution to the optic flow problem. The optical flow field is recovered by solving a system of over-determined linear equations with all the data matrices containing both outliers and noise. Here, we present a novel and very effective solution for this problem called weighted total least squares. The weights for this method are computed using a new robust statistical method named least median of squares orthogonal distances. Unlike the total least squares which is only robust to noise, this method is extremely robust to both noise and outliers and can tolerate up to 50% of equations in the system to be contaminated by outliers. The proposed weighting method is fast and the total computation remains inexpensive. To demonstrate the performance of the proposed algorithm, we compare the accuracy of our algorithm for computing optic flow field for a number of synthetic and real image sequences and show that the proposed method, despite being very simple and straightforward, out performs all methods used for comparison.

1 Introduction

In this paper, we consider the problem of finding a robust solution to a differential based optic flow problem. The differential techniques invariably involve some form of what has become known as the Optic Flow Constraint (OFC). The OFC can be written as (Horn & Schunck, 1981):

$$\frac{\partial I}{\partial x}u_x + \frac{\partial I}{\partial y}u_y + \frac{\partial I}{\partial t} = 0 \tag{1.1}$$

which relates the spatial $\partial I/\partial x$, $\partial I/\partial y$ and temporal $\partial I/\partial t$ derivatives of the image brightness function, at each point, to the optic flow (u_x, u_y) at that point. Since there is only one equation in two unknowns, it cannot be solved for both the x and y components of the optic flow, without additional assumptions or information (the well-known aperture problem). In other words, using just the information we have so far, the problem is ill-posed. Various alternative strategies to make the problem well-posed (regularize the problem) have been suggested but regardless of the strategy for overcoming the aperture problem, one usually arrives at a set of over-determined linear equations that one must solve for the optic flow at each point.

Elements of the data matrices in the final equation for solving the optic flow (which are the spatial and temporal derivatives) have to be numerically estimated, and therefore contain noise. Also, the assumptions made to overcome aperture problem (constant motion, affine motion, etc., e.g. Bergen et al., 1992) are likely to be violated due to multiple motions, transparencies, etc. Therefore, we are in fact faced with the problem of solving a set of over-determined linear equations *where all the data matrices contain both outliers and noise*. This is just one example, from many problems in computer vision, for which the solution requires solving an over-determined set of linear equations with outliers and noise in all the data matrices.

Thus, we consider the problem of finding a robust solution x to a set of over-determined linear equations:

$$Ax \approx b \tag{1.2}$$

where the data matrices A and b contain both outliers and noise (without A being rank deficient). By *robust solution*, we refer to a solution x exactly satisfying the equation $(A_s + \Delta A_s)x = (b_s + \Delta b_s)$ where the subscription s refers to the largest sub-group of equations consistent with the Least Median of Squares Orthogonal Distances (LMSOD), defined in section 2, while the Frobenius norm[1] of the perturbation matrix $\Delta = [\Delta A_s \Delta b_s]$ is kept minimum (by using the total least squares).

Finding a consistent solution to a set of over-determined linear equations Ax≈b where both of the data matrices A and b contain noise has been studied for a long time. Total least squares (TLS) is the method of choice for solving this problem (VanHuffel & Vandewalle, 1991). However, the TLS, as well as the ordinary least squares (LS) problem, is extremely sensitive to the influence of any outliers. Indeed, the breakdown point (the smallest number of contaminated data that can cause the estimator to take on values arbitrary far from true estimate, Rousseeuw and Leroy, 1987) of the TLS is only one. This means that even one contaminated element, in either of data matrices, can result in an arbitrary bad solution. It should be noted here that the TLS is often preferred over LS because the TLS solution, unlike the LS solution, which is only consistent where the observation matrix b is error free, remains consistent even when all the data matrices are noisy. Being consistent means that the estimated solution converges to the true solution as the number of equations tends to infinity (based on the assumption that all the elements of Δ are uncorrelated random variables with equal variance).

The TLS method has been frequently employed to solve many different computer vision problems. Providing a comprehensive list of all these attempts is beyond the scope of this paper but we briefly review a few relevant works. It should be noted here that these solutions have a serious limitation associated with the sensitivity of the TLS to contaminated data (outliers).

Chu and Delp (1989) have suggested using TLS for solving the set of over-determined equations resulting from an optic flow formulation. Their study addresses the rank deficient problem (where the data matrix A in the final over-determined set of linear equations is rank deficient) but fails to address the problem of having discontinuities in either the image brightness function or the optic flow itself (which commonly happens in any practical applications).

Wang et al. (1992) used the TLS to recover the smooth flow where the chances of having outliers are limited. It is important to note that, assuming the flow is smooth is not sufficient to ensure there are no outliers in the final equations. Secondly, the assumption of smooth flow, over a predefined area, is too conservative to be useful in practical applications. Weber and Malik (1995) also presented a method for estimating the optic flow based on the TLS method. In this work, the authors allow the outliers to corrupt the results when they solve the linear equations but they reject the final results

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[1] The Frobenius norm of a m x n matrix M, with entries m_{ij}, is defined as: $\|M\|_F = \sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n} m_{ij}^2}$.

based on some weighting scheme of the singular value decomposition (SVD) of the augmented data matrices. We think that this approach is too conservative. Indeed, this method under utilizes the available information by allowing the outliers to contaminate the estimate in the first place and then attempts to reject the bad results when the damage is irreversible (see section 3 for performance comparison).

Chaudhuri and Chatterjee (1991) presented a performance analysis of the TLS method for 3-D motion estimation. In this study, using synthetic data with additive uncorrelated Gaussian noise, they conclude that the TLS out performs LS method in deriving both the motion of deformable objects from range data, and the motion of a rigid object under perspective projection.

The rest of paper is organized as follows. Section 2 presents our robust TLS based solution for the optic flow problem formulated as the solution of a set of over-determined linear equations. In section 3, we calculate the optic flow for a number of image sequences and compare the results with the performance of other methods that claim high accuracy. A brief summary concludes the paper.

2. Robust TLS Based Technique

The study of robust estimators with high breakdown point, allowing a substantial portion of the data to be contaminated by outliers, has been actively researched for decades among statisticians. In 1984, Rousseeuw proposed the method of Least Median of Squares (LMedS) for the standard regression (SR) problem. LMedS has a breakdown point of 50%. Although this estimator is very robust to outliers, its theoretical performance in the presence of noise, and its computational complexity, are not attractive. Rousseeuw and Leroy (1987) proposed a very powerful method known as reweighted least squares (RLS). In this method first, a fast approximate solution to the LMedS problem is found. Then all the data points are categorized into outliers and inliers, based on their scaled residuals with respect to the LMedS solution. Finally, the regressor for the inliers is calculated using the LS technique.

Meer et al. (1991) presents a comprehensive survey on the applications of robust statistics in computer vision problems. Stewart (1997) provides a comprehensive review of the main robust estimators commonly used in computer vision literature.

The LMedS has been used (independently and concurrently) to recover the optic flow by Ong and Spann (1996a and 1996b) and Bab-Hadiashar and Suter (1996 and 1997). Although the crux of algorithms presented in both set of papers are similar, their algorithms differ in the way that the optic flow fields are calculated. To resolve the issue of multiple motions, Ong and Spann compare the flow computed for a block with the flow computed for a shifted (right and downward) block. The estimate from the block with the greatest number of inliers is retained. Although this technique may prove to be effective for occluding objects, it cannot resolve transparent motion. Bab-Hadiashar and Suter however, use a small patch centered at every pixel and employ a robust LMedS based estimator to reject the OFC belonging to pixels with motions widely different from the majority of pixels in the original patch. Then, they solve all the remaining OFC equations (inliers) using the LS technique. Moreover, the Bab-Hadiashar and Suter approach has an extra stage that performs a reliability check. Although we follow the Bab-Hadiashar and Suter's approach for calculating the optic

flow (and we compare our results with their results), the presented work here goes well beyond the common basic ideas developed in both set of papers.

In this paper, we propose the Weighted Total Least Square (WTLS) method, similar to the RLS method proposed by Rousseeuw and Leroy (1987), for solving an over-determined set of linear equations A$x \approx b$ where the data matrices A and b contain both outliers and noise without A being rank deficient. The proposed method differs from the RLS method in two ways. Firstly, unlike the RLS method, the outliers (of equation 1.2) in this method are detected using the LMSOD technique. LMSOD seeks an approximate solution x which exactly satisfies $(A+\Delta A)x = b+\Delta b$ while minimizing the median of squares orthogonal distances between solution x and the geometrical entity (line, plane, hyper-plane, etc) represented by every equation in the original set. The second major difference is that in the WTLS method, unlike the RLS method which uses ordinary least square to solve the inlier group, the total least square technique is used to solve the remaining system of over-determined linear equations (after rejecting the outliers).

It is important to note here that this method has all the advantages of the TLS method without being sensitive to the influence of outliers. Comparing LMSOD to the LMedS, it is trivial to show that LMSOD method also has the breakdown point of 50%.

2.1 Proposed algorithm

For the sake of clarity, we describe the proposed algorithm in six steps:

1. Estimate the spatio-temporal derivatives of the image brightness function. We choose to use, for our experiments, convolutions with derivatives of Gaussian functions with equal spatial and temporal standard deviations (Nagel, 1995).
2. Select a patch of the image, over which we are going to assume some motion consistency. The precise form of the motion consistency is not essential: we are simply assuming a single or dominant population (we only recover the dominant population if there is more than one - our method can be elaborated to remove the dominant population and re-solve for any secondary populations). In this paper, we restrict the motion consistency to one of two forms: constant motion and affine motion with in a patch.
3. Use a fast and robust approximate LMSOD solution to obtain a temporary estimate of the solution x (free from the influence of any existing outlier). Here, we propose a method similar to the one presented by Rousseeuw and Leroy (1987). The method starts by randomly choosing a group of sample equations. Each sample must group contain n equations where n is the number of rows in the solution matrix x (equation 1.2). Moreover, all the chosen equations must be independent (to ensure the existence and uniqueness of the solution). By solving every such set of equations and finding the median of the squared orthogonal distances between this solution and the rest of the equations in the original set, one can find the solution, which approximately satisfies the LMSOD.

   Similar to the LMedS case, one needs to choose only one sample group of size n that belongs to majority, in order to return the approximate solution associated with the majority. Therefore only a small number of sample groups is required to have the probability of having at least one good sample close to 1 (Rousseeuw & Leroy, 1987).

4. Having found the approximate LMSOD, one can weight the different equations based on the vertical distance between the LMSOD solution and the geometrical entity (line, plane, hyper-plane, etc.) represented by every equation in the original system of equations. Here, we closely follow the recommendations made by Rousseeuw and Leroy (1987) for scaling the residuals in two steps. The detail of the weighting scheme is described in appendix. We then identify the outliers by comparing the scaled residuals with some constant threshold. After identifying the outliers, we eliminate the outlier equations (weight them by zero) to arrive at a new system of over-determined linear equations $A_s\ x = b_s$ in which the number of equations are now less than or equal the original set.
5. The final solution can be obtained by solving the new system of over-determined linear equations using the total least squares technique (VanHuffel & Vandewalle, 1991):

$$x = (A_s{}^T A_s - \kappa^2 I)^{-1} A_s{}^T b_s \tag{2.1}$$

where κ is the smallest singular value of the augmented matrix $[A_s\ b_s]$ and I is the identity matrix.
6. This last step in this algorithm is to ensure that the final solution is acceptable. Thus, a measure of reliability is proposed here to examine the validity of the end results. The detail of this step is described in the following section.

2.2 Measure of Reliability

Although the LMSOD techniques have the highest possible breakdown point (50%) of all known robust estimators, it has the potentially fatal flaw in that is still produces an estimate, even if the number of outliers is more than 50%. Moreover, there are extreme cases where an image patch may not contain sufficient data (lack of texture) or data so badly corrupted (aliasing for example) for any estimate to be valid. Thus we still need to validate the estimate produced by our method. A tool for the validation process can be modeled on "the coefficient of determination" (Kvalseth, 1985). The coefficient of determination, denoted R^2, has been defined for the Standard regression problem in at least nine different ways. However, although we are guided by analogy with the SR problem, we are interested in robust forms of TLS. We define our own measure, which is also called R^2, similar to the one presented by Bab-Hadiashar and Suter (1997). For the WTLS technique, we want to ensure that the Frobenius norm of the perturbation matrix $\Delta=[\Delta A_s\ \Delta d_s]$ is small enough for the solution to be acceptable. Since it has been shown that κ (the smallest singular value of the augmented matrix $[A_s\ d_s]$) is equal to the Frobenius norm of the perturbation matrix Δ for the calculated x (VanHuffel & Vandewalle, 1991), we propose the following R^2 statistic:

$$R^2_{WTLS} = 1 - \frac{\kappa^2}{\sum_i (d_{si} - \bar{d}_{si})^2} \tag{2.2}$$

where d_{si} represents the different elements of vector d_s and maximum number of i is set by the number equations regarded as inliers.

3 Optic Flow Computation

To evaluate the performance of the proposed estimator for recovering the optic flow field, we compute the flow field and the error statistics for a few synthetic and real

image sequences whose the "ground truth" motion is known. As mentioned in section 1, a common approach to the optic flow problem is to formulate the flow field as a solution to an over-determined system of linear equations (similar to equation 1.2). The number of unknowns in this approach (number of columns in matrix A) depends on the model of motion in every patch of the image. Constant (2 unknowns) and affine (6 unknowns) motions are the most common models of motion proposed in the optic flow literature. To keep the computation minimum, we first solve the LMSOD (step 3) for all the OFC contained in a square window with constant model of motion. Then, we calculate the weights for every OFC based on its residual with respect to LMSOD. We simply reject the constraints, whose scaled residual is above some threshold (step 4). The final steps in estimating the flow field are to solve the new system of over-determined linear equations using total least squares and compute the associated R^2 statistics. To improve the accuracy at the last stage, we solve the weighted set of OFC using an affine motion model (six unknowns). The idea behind this is very simple. The outliers contaminating the OFC (due to multiple motions, transparency, etc) are essentially independent of the motion model and by rejecting the outliers using constant motion model, the computational time is reduced. It is important to note that this argument is only justified for small windows where the chance of disregarding good points at the tail of the affine model by the robust solution calculated using constant model is negligible. One, of course, may achieve slightly better results by using the affine model of motion in both steps.

3.1 Synthetic and Real Image Sequences

To demonstrate the effectiveness of our method for dealing with motion boundaries, we use a number of synthetic and real image sequences as benchmark. These image sequences are: New-Sinusoid1, Yosemite and Otte (see Bab-Hadiashar, 1997 for a detailed description of each image sequence). The error statistics related to each image sequences and their comparison with a number of other methods is shown in tables 3.1 to 3.3.

These results show that our method is very robust to the existing depth and motion discontinuities. It can be seen from these tables that our WTLS based method outperforms other methods used in comparison.

4 Conclusion

This paper presents a novel method for solving a system of over-determined linear equations when the parameters of the equations are contaminated with both outliers and noise. The solution to this type of problem is frequently sought in the study of different computer vision problems. The proposed algorithm uses a new method named the least median of squares orthogonal distances combined with the well-known total least squares for dealing with the outliers and noise, respectively. A fast method for computing an approximate solution to the LMSOD is also proposed which makes the computation inexpensive. The performance of this method has been demonstrated by solving the optic flow problem. Although the presented algorithm is conceptually very straight forward, it out-performs any other (often very sophisticated) optic flow technique.

Technique	Avg. Error	Std. Dev.	Density
Fleet and Jepson(σ=2.5, τ=1.25)	7.39°	10.84°	43.4%
Fleet and Jepson(σ=2.5, τ = 2.5)	1.41°	3.65°	46.0%
WLS2(σ=1.0,5x5,m=30,without check)	1.56°	7.12°	100%
WLS2(σ=1.0,5x5,m = 30, R^2 = 0.9999)	0.05°	0.06°	84.6%
WLS6(σ=1.0,5x5,m=30,without check)	1.51°	5.86°	100%
WLS6(σ=1.0,5x5,m = 30, R^2 = 0.9999)	0.05°	0.06°	83.5%
WTLS2(σ=1.0,5x5,m=30,without check)	2.82°	8.82°	100%
WTLS2(σ=1.0,5x5, m = 30, R^2 = 0.9999)	0.05°	0.06°	76.1%
WTLS6(σ=1.0,5x5,m=30, without check)	1.51°	6.23°	100%
WTLS6(σ=1.0,5x5, m = 30, R^2 = 0.9999)	0.08°	0.22°	88.4%

Table 3.1: Error analysis using New-Sinusoid1 image sequence.

Technique	Avg. Error	Std. Dev.	Density
Fleet and Jepson (σ = 2.0, τ = 1.25)	2.08°	3.77°	50.6%
Fleet and Jepson (σ = 2.0, τ = 2.50)	2.56°	4.08°	57.1%
Fleet and Jepson (σ = 2.5, τ = 1.25)	2.05°	3.85°	55.8%
Fleet and Jepson (σ = 2.5, τ = 2.50)	2.53°	4.25°	62.2%
Giachetti and Torre (1996)	5.33°	-°	100(25)%
WLS2(σ=2.0,15x15,m=30,without check)	3.39°	6.55°	100%
WLS2(σ=2.0,15x15,m=30,R^2 = 0.99)	1.50°	2.22°	59.1%
WLS6(σ=2.0,15x15,m=30,without check)	3.51°	6.48°	100%
WLS6(σ=2.0,15x15,m=30,R^2 = 0.99)	1.44°	1.92°	55.9%
WTLS2(σ=2.0,15x15,m=30,without check)	3.74°	8.09°	100%
WTLS2(σ=2.0,15x15,m=30,R^2 = 0.99)	1.61°	2.60°	71.2%
WTLS6(σ=2.0,15x15,m=30,without check)	3.67°	7.37°	100%
WTLS6(σ=2.0,15x15,m=30,R^2 = 0.99)	2.46°	4.71°	82.0%
WTLS6(σ=2.0,15x15,m=30,R^2=0.999)	1.55°	2.34°	51.6%

Table 3.2: Error analysis using Otte image sequence (Otte & Nagel, 1994).

Technique	Avg. Error	Std. Dev.	Density
Fleet and Jepson (σ = 1.5, τ = 1.25)	4.95°	12.39°	30.6%
Fleet and Jepson (σ = 1.5, τ = 2.5)	4.29°	11.24°	34.1%
Weber and Malik (1993)	3.42°	5.35°	45.2%
Szeliski and Coughlan (1994)	3.06°	7.54°	39.6%
Weber and Malik (1995)	4.31°	8.66°	64.2%
Giachetti and Torre (1996)	2.82°	6.98°	70.79%
WLS2(σ=2.0,15x15,m = 30, without check)	3.17°	6.46°	100%
WLS2(σ=2.0,15x15,m = 30, R^2 = 0.99)	3.13°	7.07°	76.2%
WLS6(σ=2.0,15x15,m = 30, without check)	2.86°	6.76°	100%
In the following results, the cloud region is not included.			
Black (1994)	3.52°	3.25°	100%
Black and Jepson (1994)	2.29°	2.25°	100%
Black and Anandan (1996)	4.46°	4.21°	100%
Ju et al (Skin & Bones, 1996)	2.16°	2.00°	100%
WLS2(σ=2.0,15x15,m=30,without check)	2.51°	2.57°	100%
WLS6(σ=2.0,15x15,m=30,without check)	2.02°	2.05°	100%
WTLS2(σ=2.0,15x15,m=30,without check)	2.56°	2.34°	100%
WTLS6(σ=2.0,15x15,m=30,without check)	1.97°	1.96°	100%

Table 3.3: Error analysis using Yosemite image sequence.

The first column of entries determines the method applied to generate the row of error statistics. In our method (WTLS) the numbers 2 and 6 represent the constant and affine motion models, respectively. The numbers in brackets depict the size of the Gaussian smoothing (σ is the standard deviation of the filter), the size of local patch used (p), the number of pairs of lines used to approximate the LMedS or the LMSOD (m), and the reliability threshold (R^2), in that order.

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Appendix: Residual Scale and Outlier Threshold

In our method, having obtained an approximate solution, based on an approximate LMSOD, we wish to assess the reliability of each equation. The following procedure, which is similar to the recipe proposed by Rousseeuw and Leroy (1987), is used for detecting outliers.

We first calculate, for each equation in the original system of linear equations, a residual r_i by finding the distance between the LMSOD solution and the geometrical entity (line, plane, hyper-plane, etc.) represented by that equation. Then we calculate a scale factor s^0 according to:

$$s^0 = 1.4826\left(1+\frac{5}{p-2}\right)\sqrt{\operatorname*{med}_i r_i^2} \qquad \text{(A.1)}$$

where p is the number of equations in the original system of linear equations.

We then associate a binary weight w_i so that the weight is 0 for any constraint whose residual r_i is such that absolute of r_i/s^0 is greater than 2.5 (and the weight is otherwise equal to 1).

Rather than using these weights to directly reformulate the problem now as a (weighted) Total Least Squares problem, we go through one more step of scaling. This is because the original weights were chosen, according to equation (A.1), using the median *involving the outliers*. Since we now have a better idea of which are truly outliers, we calculate:

$$\sigma^* = \sqrt{\frac{\sum_{i=1}^{p} w_i r_i^2}{\sum_{i=1}^{p} w_i - p}} \qquad \text{(A.2)}$$

and we, finally, reject those constraints for which the associated absolute value of r_i/s^0 is greater than 2.5.