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Motivation

- Different meanings of “good” PCB:
 - the circuit works as expected,
 - the circuit is not disturbed by external events/noise (fluorescent lamp, relay switching, CRT monitor, mobile phone, etc.),
 - the circuit is not disturbing its surrounding (radio receivers, etc.),
 - the circuit passes applicable ESD and EMC standards (both conducted and radiated),
 - the circuit provides clean measuring possibilities.
- Different boards are designed under completely different aspects (e.g. ATE handler adapter vs. chip-level EMC compliance test PCB).
- Reciprocity principle: a circuit that is emitting at a given frequency, is likely to be susceptible for external disturbances at this frequency, i.e. broadband noise generators often prove to be sensitive.

RLC theory I – Resonance frequency II

- The damping constant of a naturally damped oscillation is given as:

$$\beta_{\text{series}} = \frac{R}{2L} \quad \beta_{\text{parallel}} = \frac{1}{2R \cdot C}$$

- Definition of the logarithmic decrement Λ : $\Lambda = \ln \left(\frac{V_{\text{peak \#i}}}{V_{\text{peak with same sign \#i+1}}} \right)$

- Quality factors: $Q_{n,\text{series}} = \frac{\omega_n \cdot L}{R}$ $Q_{n,\text{parallel}} = \omega_n \cdot R \cdot C$

- For both series and parallel tank the following holds:

$$\Lambda = \frac{\pi}{Q_n} = \beta \cdot \frac{2\pi}{\omega_n} \quad \omega_n^2 = \omega_0^2 - \beta^2 \quad Q_n = \frac{\omega_n}{2\beta} = \frac{\pi}{\Lambda} = \sqrt{Q_0^2 - \frac{1}{4}}$$

- The 3dB bandwidth is defined as follows:

$$B = |f_2 - f_1| = \frac{f_0}{Q_0} \text{ with } f_1 \text{ and } f_2 \text{ defined as}$$

$$|Z(f_1)| = |Z(f_2)| = Z(f_0) / \sqrt{2} \text{ for parallel tank}$$

$$|Z(f_1)| = |Z(f_2)| = Z(f_0) \cdot \sqrt{2} \text{ for series tank}$$

$$\text{for both tanks } f_0^2 = f_1 \cdot f_2$$

RLC theory I – Resonance frequency IV

- Naturally decaying series tank with $C=100\text{nF}$, $L=1\mu\text{H}$, $R=2\Omega$ and the capacitor initially charged to V_{C0} : $f_0=503.3\text{kHz}$, $f_n=477.5\text{kHz}$, $Q_0=1.58$, $Q_n=1.50$, $\Lambda=2.09$, $\beta=10^6\text{ 1/s}$
- Notice the 5.1% frequency and Q deviation from the undamped oscillation case.
- The current and the capacitor voltage are given with $\tan(\varphi)=\omega_n/\beta$, $\varphi=71.6^\circ$:

$$i(t) = -V_{C0} \cdot C \cdot \frac{\omega_0^2}{\omega_n} \cdot e^{-\beta \cdot t} \cdot \sin(\omega_n \cdot t) \qquad v_C(t) = V_{C0} \cdot \frac{\omega_0}{\omega_n} \cdot e^{-\beta \cdot t} \cdot \sin(\omega_n \cdot t + \varphi)$$

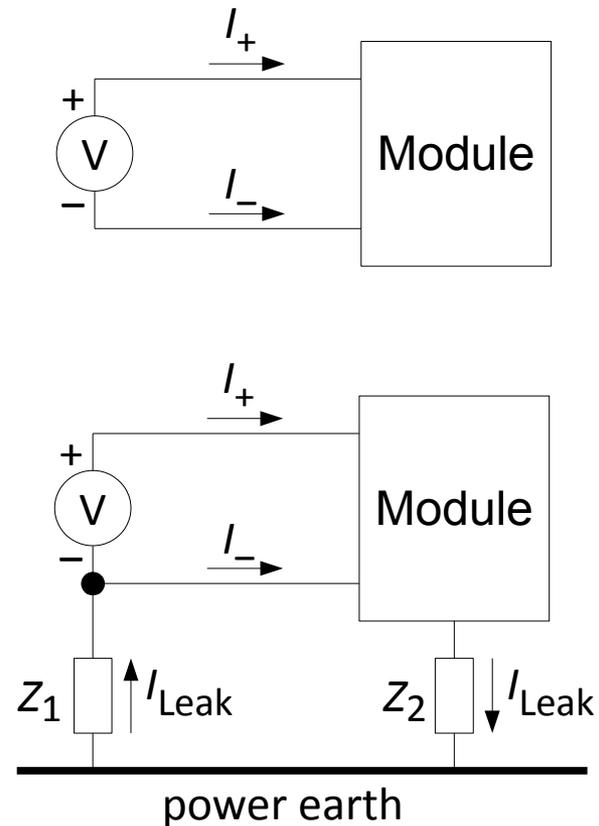
- Naturally decaying parallel tank with $C=100\text{nF}$, $L=1\mu\text{F}$, $R=5\Omega$ and the capacitor initially charged to V_{C0} : $f_0=503.3\text{kHz}$, $f_n=477.5\text{kHz}$, $Q_0=1.58$, $Q_n=1.50$, $\Lambda=2.09$, $\beta=10^6\text{ 1/s}$
- Notice the identical values to the series tank case (see *RLC theory III – Series vs. parallel notation II* for details).
- The capacitor current and voltage are (with $\varphi=71.6^\circ$, as for the series tank):

$$i_C(t) = V_{C0} \cdot C \cdot \frac{\omega_0^2}{\omega_n} \cdot e^{-\beta \cdot t} \cdot \sin(\omega_n \cdot t - 2\varphi) \qquad v(t) = -V_{C0} \cdot \frac{\omega_0}{\omega_n} \cdot e^{-\beta \cdot t} \cdot \sin(\omega_n \cdot t - \varphi)$$

- Notice the identical formulas for series and parallel tanks except for the additional -2φ in the sinus argument and the flipped sign.

Common mode and differential mode signals I

- Theory distinguishes common mode (CM) and differential mode (DM) signals.
- Easier to analyze and to take countermeasures → CM and DM filters.
- Ideal case $I_+ + I_- = 0$. Unfortunately, this doesn't happen in real life.
- Real case $I_+ + I_- = I_{\text{leak}}$
- By definition
$$I_{\text{CM}} = I_+ + I_- \quad \text{and}$$
$$I_{\text{DM}} = I_+ - I_-$$
- Solving the equation system yields
$$I_+ = I_{\text{CM}}/2 + I_{\text{DM}}/2 \quad \text{and}$$
$$I_- = I_{\text{CM}}/2 - I_{\text{DM}}/2$$



Single-ended and differential signals II

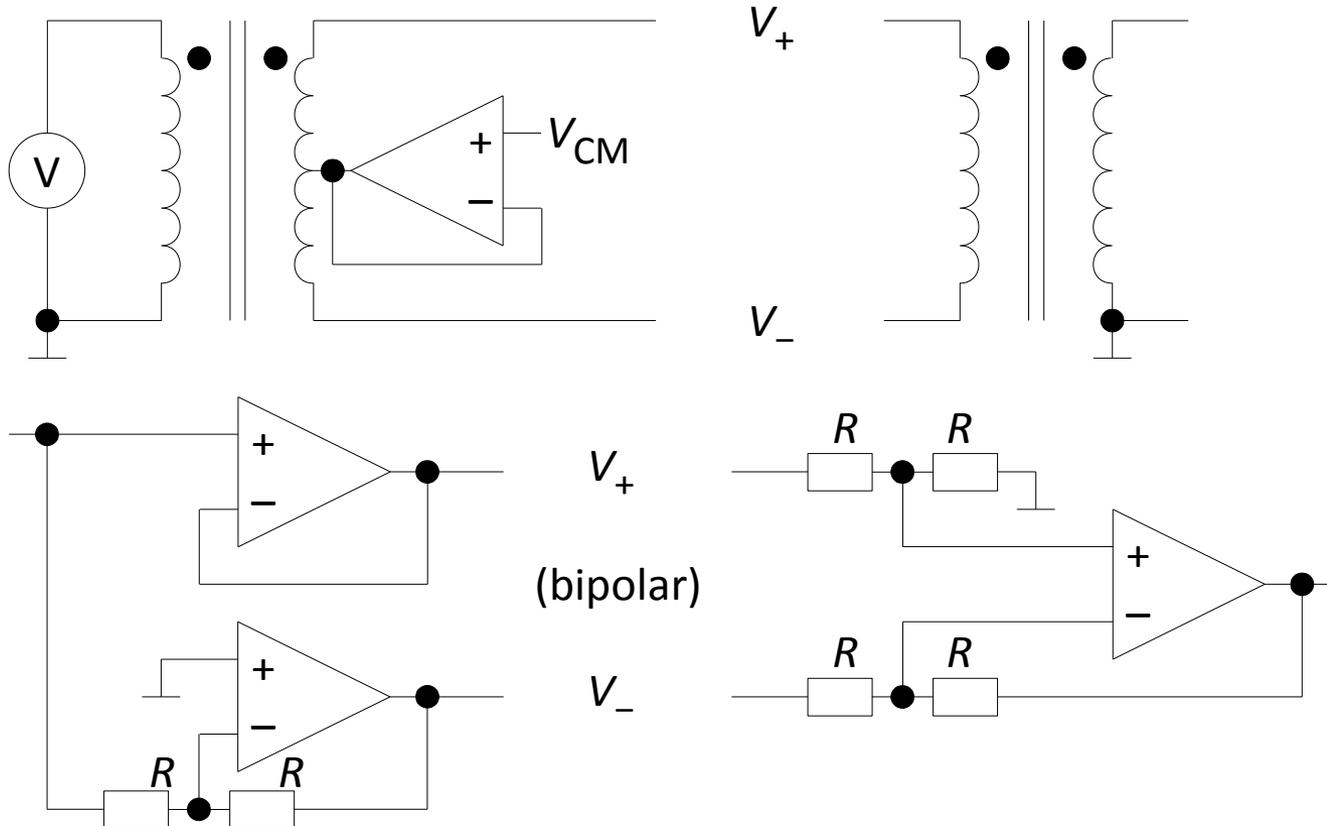
- What is better: a differential signal or a (possibly shielded) single-ended signal?
- For coupling we're interested in the reactive near field, as the distance from the cable/track is less than $\lambda/(2\pi)$, roughly 48cm (19") free space for 100MHz.
 - A differential signal in twisted pair (shielded or unshielded) cable (STP/UTP) has ideally zero near field, if the twisting is $<\lambda/10$. For untwisted parallel PCB tracks the fields don't cancel perfectly, so they have both capacitive and magnetic coupling to neighbor tracks.
 - Even a non-sinusoidal single-ended signal in a coax or twisted pair cable has ideally zero near field, as long as no leakage currents exist. Otherwise time- and position-dependent near field exists.
- The input supply voltage ripple generated by a differential driver is ideally zero.
- If analog unipolar differential signals are required, depending on the load, an adequate common mode voltage (e.g. $V_{DD}/2$) with large current source/sink capability may be needed.
- A single-ended coax cable is far the easiest to handle, if the application does not require differential signaling (use STP in the latter case for best EMC performance).

Single-ended and differential signals III

- An analog wideband single-ended to differential converter (balun=balanced-to-unbalanced) can be realized by a center-tapped transformer with $n:n$ windings. Of course, signal transformation by different primary and secondary winding count is also possible.
 - The center-tapped transformer usually provides better matching than two separated secondary windings. But even this structure may need manual symmetry tuning.
 - Check commercially available baluns/transformers (“stromkompensierte Spulen”) for their imbalance properties.
 - If common mode voltage is not an issue, the center tap can be simply grounded. Otherwise, the common mode voltage can be set in a convenient way by a voltage source with low output impedance.
 - Alternatively, two OpAmps can be used: one in inverting, the other in non-inverting configuration. For better matching, do not omit the non-inverting OpAmp.
- Differential to single-ended conversion can be done by a simple transformer.
 - Alternatively, an OpAmp can be used with four matched resistors. If the input impedance has to be high, an instrumental amplifier (InA) shall be used.

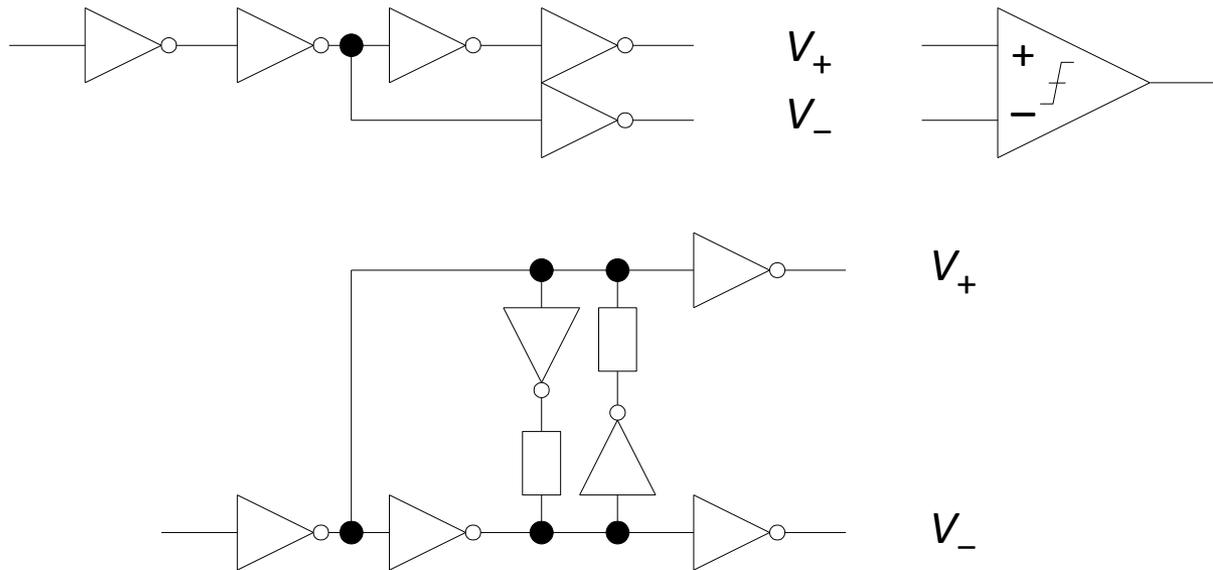
Single-ended and differential signals IV

- Analog single-ended to differential conversion and vice versa.
 - Usually, trimming is required for the resistors.
 - For single-ended to differential: use a dual-OpAmp device for matching reasons.



Single-ended and differential signals V

- For unipolar digital signals a fast inverter and a comparator can be used. Do not omit the input (leftmost) inverters, they reshape the signal edges and decouple the differential signal noise from the input. Use a hexa inverter device for matching reasons.
- If the delay mismatch is critical, use a hexa inverter device for matching reasons and the lower circuitry. This minimizes the phase error, but has a high current consumption at the signal edges. Select the resistor value to about five times the inverter output resistance. Resistor selection/tuning shall be done by watching the signals on an oscilloscope.
- The center-tapped transformer and a dual-inverter for the differential output signal can also be used.



EM theory basics I – Inductance confusion I

- Different kinds of inductance: self, mutual, loop inductance (to complicate things, all of them are frequency dependent) in addition to internal and external inductance.
- Self inductance is the inductance of a conductor #1 defined by $L_{11} = \frac{\int A_1 \cdot J_1 \cdot dV_1}{I_1^2}$ with A_1 as the magnetic potential caused by current I_1 flowing in the conductor; J_1 the current density and dV_1 the differential volume element of the conductor.
- Mutual inductance describes the inductive coupling between two conductors. By definition the current direction is the same in both conductors for positive mutual inductance. Note, that for our practical cases $L_{21}=L_{12}$. Mutual inductance for perpendicular conductors is ideally zero. $L_{21} = \frac{\int A_1 \cdot J_2 \cdot dV_2}{I_1 \cdot I_2}$
- Loop inductance is the complete inductance of two conductors, measurable at one end under the condition $I_1+I_2=0$ (e.g. the two conductors are shorted at the other end). Due to opposite current direction in the conductors: $L_{21}=L_{12}<0$.

$$L_{\text{loop}} = L_{11} - L_{12} - L_{21} + L_{22}$$

EM theory basics I – Inductance confusion II

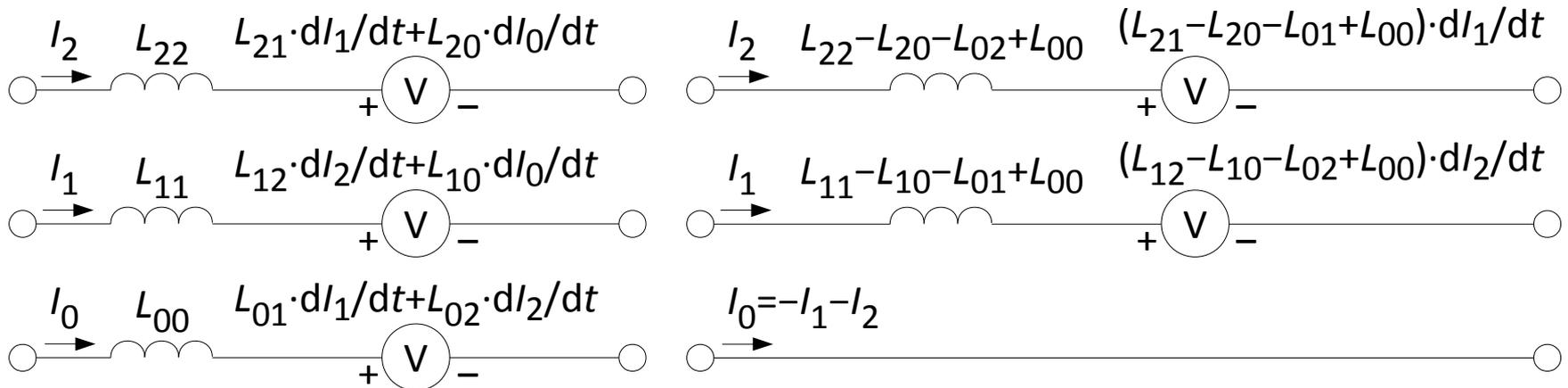
- Self inductance can be separated into internal and external inductance.
 - Internal inductance is attributed to the magnetic field within the conductor,
 - external inductance is attributed to the magnetic field external to the conductor.

$$L_{11} \equiv L_{\text{self}} = L_{\text{int}} + L_{\text{ext}}$$

- The magnetic field outside the conductor does not change with skin effect, but the field within the conductor is reduced (the center of the conductor is magnetic field free for high frequencies). This reduces the internal inductance with frequency.
- For rectangular cross section with length b , width w and height h and a circular cross section conductor:
$$L_{\text{int,rect}} = \frac{\mu \cdot b \cdot w}{12 \cdot h} \approx \frac{w}{h} \cdot 1 \text{ nH/cm} \quad L_{\text{int,circ}} = \frac{\mu \cdot b}{8 \pi} = 0.5 \text{ nH/cm}$$
- The internal inductance at DC is about 20...40% of the self inductance. At very high frequencies the internal inductance diminishes, so the self inductance remains about 60...80% of the DC value.
- Note, that German literature often talks about “Selbstinduktivität”, when they really mean loop inductance (e.g. in case for inductance formulas of coax cable, double conductor in free space, or single conductor above ground plane configurations).

EM theory basics I – Inductance confusion III

- Multiple descriptions exist for a given physical setup. Three conductor physical setup (left) showing self inductance L_{ij} and mutual inductance L_{ij} . Loop inductance representations with ideal reference conductor (right) showing the loop inductance $L_{ij} - L_{i0} - L_{0j} + L_{00}$ and the loop mutual inductance $L_{ij} - L_{i0} - L_{0j} + L_{00}$
- The right model requires $I_0 + I_1 + I_2 = 0$. For details see “Simulating the W element” in the Intranet HF & Transponder competence center's PDF archive.



EM theory basics II – High frequency effects I

- Every piece of conductor has parasitic effects. Rules of thumb:
 - coax cable & 50Ω track: 1pF/cm capacitance, narrow track & twisted pair cable: 0.5pF/cm,
 - coax cable & 50Ω track: 2.5nH/cm loop inductance, twisted pair & narrow track: 10nH/cm loop inductance,
 - track/cable introduces 50ps/cm signal delay,
 - self-inductance decreases somewhat with frequency, resistance rises significantly with frequency (complex skin effect): $Z(f) = Z_{DC} + z \cdot \sqrt{f}$
- *RLC* elements exist to neighbor structures, too.
- Physical *R/L/C* components all have frequency-, temperature- and voltage-dependent *RLC* equivalent circuits and self-resonance.
 - 0805 capacitors have typically 1.5nH self inductance and 0.15Ω resistance,
 - 0805 pads have 0.2pF capacitance to ground (relevant for capacitors < 10pF).
 - See also Practical issues XII – Tolerances and aging.

EM theory basics II – High frequency effects II

- Skin effect and skin depth as a mathematical aid

- Skin depth is $\delta(f) = \sqrt{\frac{\rho}{\pi \cdot f \cdot \mu}}$ with ρ : specific resistance and μ : permeability

- For 35 μ m copper tracks no skin effect up to 15MHz ($\mu_r=1$, $\rho_{Cu}=1.8e-8\Omega m$).

- 10mil (0.25mm) track centered 0.32mm (12.6mil) above a 100mm (4") wide ground plane (both layers 35 μ m thick): loop resistance (including ground plane resistance – however, this is still below 1m Ω /cm @ 1GHz) and loop inductance:

- $R_{DC}=20m\Omega/cm$, $R_{100MHz}=76.4m\Omega/cm$, $R_{1GHz}=255m\Omega/cm$

- $L_{DC}=10.94nH/cm$, $L_{100MHz}=6.15nH/cm$, $L_{1GHz}=6.06nH/cm$

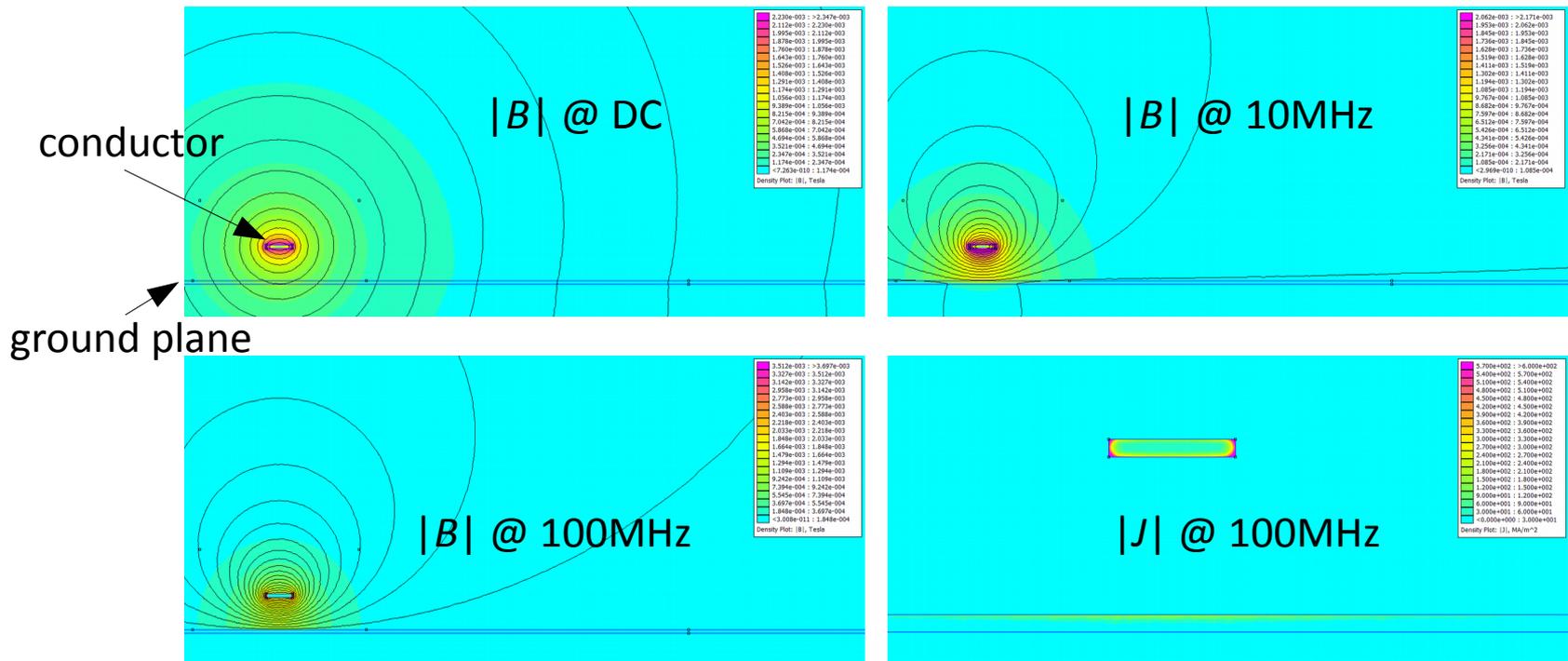
- 0.14mm² cross section (AWG26) copper twisted pair cable in free space (loop resistance and loop inductance). Skin effect occurs here above 100kHz (!):

- $R_{DC}=2.6m\Omega/cm$, $R_{100MHz}=53.4m\Omega/cm$, $R_{1GHz}=111.5m\Omega/cm$

- $L_{DC}=6.35nH/cm$, $L_{100MHz}=5.10nH/cm$, $L_{1GHz}=5.04nH/cm$

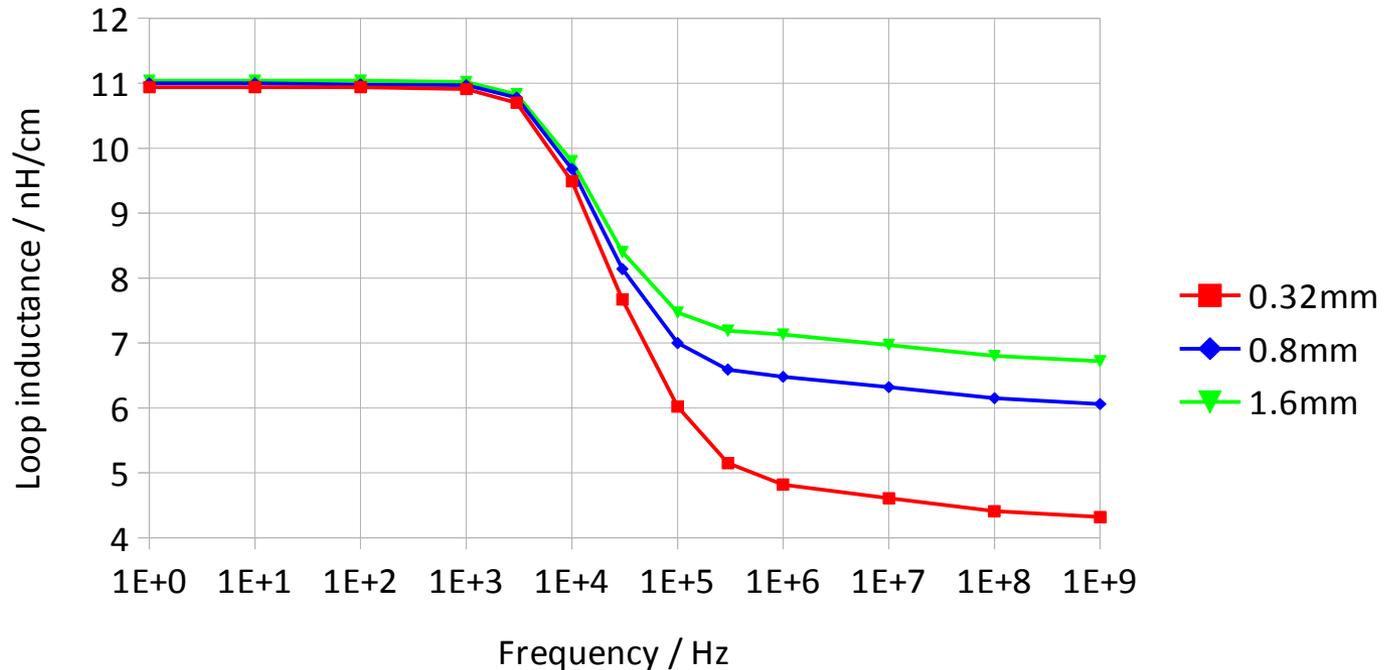
EM theory basics II – High frequency effects III

- 10mil (0.25mm) wide and 35 μ m thick conductor 0.32mm above ground plane. Magnetic flux $|B|$ at DC, 10MHz and 100MHz and current density $|J|$ at 100MHz. A magnetic field concentration can be observed with rising frequency.
- Note, that the skin effect for the ground plane means, that the return current is concentrated below the conductor.



EM theory basics II – High frequency effects IV

- 10mil (0.25mm) wide and 35 μ m thick conductor above 4" (100mm) wide ground plane loop inductance frequency dependence for different insulator thickness.
- Per unit length loop inductance reduction is due to field concentration around the conductor at high frequencies.
- Textbook loop inductance formulas are given for the DC case.



EM theory basics II – High frequency effects VI

- Characteristic impedance

- Every trace/cable has a geometry dependent characteristic impedance, at which no reflection occurs, if terminated accordingly at both ends. L' is the per unit length loop inductance, C' is the per unit length capacitance.

$$Z_0 = \sqrt{\frac{L'}{C'}}$$

- This impedance is not a physical resistance by means of thermal noise and signal damping. That is, a lossless/noiseless superconducting cable and free space also have a meaningful characteristic impedance. For example, free space has

$$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377 \Omega \equiv \frac{E}{H}$$

- Radiation – from Maxwell's equations: if it is not DC, it is radiating.
 - Power loss into free space: KCL and KVL (Kirchhoff I-II) and Ohm's Law not valid any more – derived for stationary current fields, may be used with caution only.
 - Radiation especially occurs at 90° corners a.k.a. kinks (a via is a kink, too) and at special geometries, such as stubs, slots and patches.
 - Antenna structures work identically in both directions (transmitting and receiving).

Control theory basics I – Transfer function II

- Transfer functions (TF) in the frequency domain can be written as a ratio of two polynoms:

$$G(s) = \frac{\sum_{i=1}^n a_i \cdot s^i}{\sum_{i=1}^m b_i \cdot s^i} \quad \text{with } n \leq m$$

- The roots of the nominator are called zeros. The roots of the denominator are called poles.
- According to polynom decomposition, both zeros and poles are either real numbers or conjugate complex. Allowing for the zeros z_i and the poles p_i complex numbers:

$$G(s) = k_0 \cdot \frac{\prod_{i=0}^n (s - z_i)}{\prod_{i=1}^m (s - p_i)}$$

Control theory basics I – Transfer function IV

- The poles of the closed loop are *not identical* to the poles of the open loop.
- Decomposition of the $A(s)$ and $B(s)$ functions as a polynom ratio shows $G(s)$ in a different light (omitting the function argument s for brevity):

$$\text{with } A = \frac{n_A}{d_A} \text{ and } B = \frac{n_B}{d_B}$$

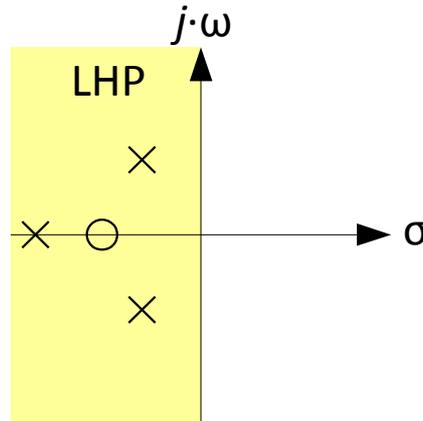
$$H = A \cdot B = \frac{n_A \cdot n_B}{d_A \cdot d_B}$$

$$G = \frac{A}{1 + A \cdot B} = \frac{n_A \cdot d_B}{n_A \cdot n_B + d_A \cdot d_B}$$

- The feedback shifts poles (especially, if we have zeros in the open loop TF): the open loop characteristic polynom is $d_A \cdot d_B$, but in the closed loop we get a new term with the product of the open loop zeros.
- By inspecting the nominators of H and G : the feedback replaces the zeros of the feedback function by the poles of the feedback function.

Control theory basics II – Pole zero plot I

- The poles and zeros can be plotted graphically on the complex (angular) frequency plane. A typical plot shows zeros (circles) and poles (crosses) in the left half plane (LHP). A pole-zero plot with k_0 completely describes a SISO LTI system without dead time.



- Such a system is stable if and only if: all poles are lying in the LHP
- Such a system is unstable if: there are poles in the right half plane (RHP) or there is a multiple pole at the origin ($s=0$).
- Such a system is at its stability boundary if there are single poles at $\sigma=0$.
- Such a system has minimal phase, if all zeros are in the LHP. The system has nonminimal phase, if some zeros are in the RHP (e.g. SEPIC converter).
- The quality factor of a conjugate complex pole is $p=\sigma_p+j \cdot \omega_p$ given as: $Q_p=|\omega_p|/|p|$

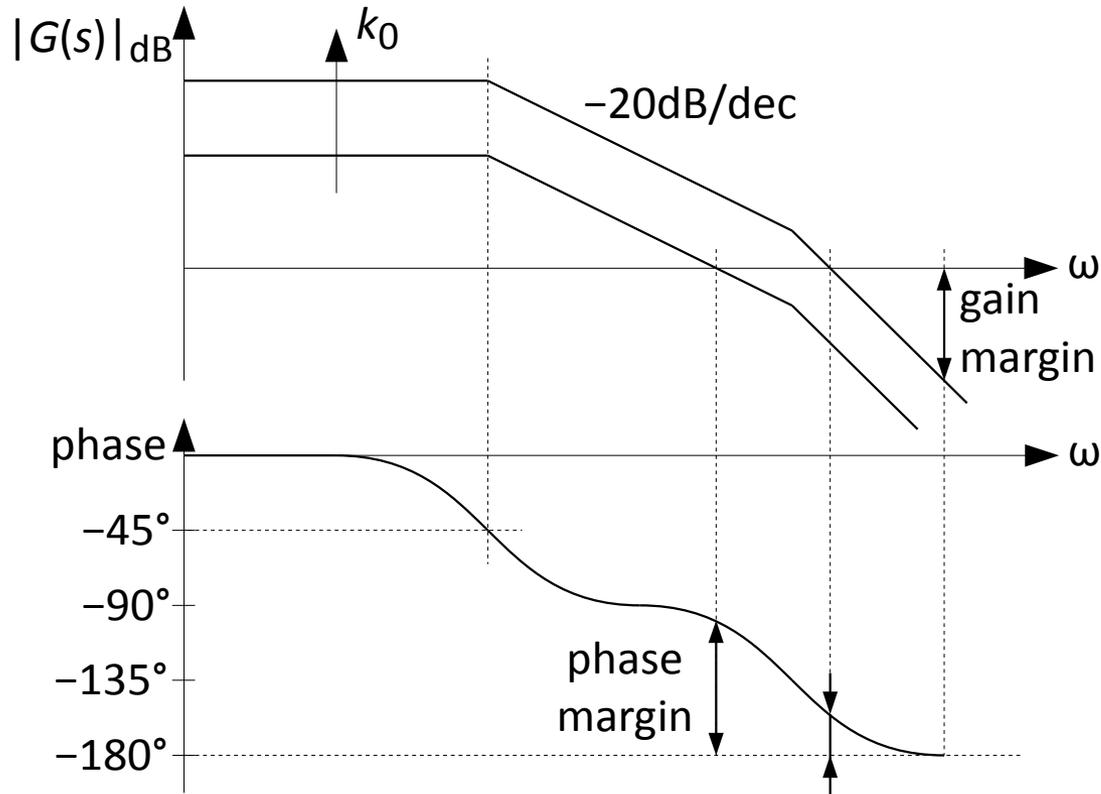
Control theory basics II – Pole zero plot II

- Low frequency (dominating) poles and zeros are close to the $\sigma=0$ axis.
- High frequency poles and zeros lie towards the left end.
- The TF is a complex function describing magnitude and phase.
- A LHP pole decreases the phase (more negative).
- A LHP zero increases the phase.
- Attention: a RHP zero *decreases* the phase.
- Negative feedback shifts the phase by 180° .
- The stability boundary is given for $|G(s)|=1$ and $\text{phase}(G(s))=0^\circ=-360^\circ$
- Accounting for the negative feedback, the maximum allowed phase shift of the open loop is -180° if stability is required.
- If a conjugate complex pole is dominating, the ringing is getting more severe with increasing Q_p . Considering the step answer and allowing for 4.3% overshoot, $Q_p < 0.7$ is required (that is, for equally scaled σ and $j\cdot\omega$ axis, less than 45° angle from the σ axis).

Control theory basics III – Bode plot I

- The Bode plot shows the magnitude (in dB) and the phase of a transfer function.
- The *closed loop* is unconditionally stable if and only if the Bode plot of the *open loop* features *all* of the following:
 - gain margin (the magnitude is below 0dB @ phase= -180°)
 - phase margin (the phase is above -180° @ magnitude=0dB)
 - the magnitude crosses the 0dB point with -20dB/decade slope
 - all the Bode plot “wisdom” is valid only for minimal phase systems
- The last two conditions are often ignored with catastrophic consequences. If the slope at the 0dB point is -40dB/decade , the ringing of the closed loop can be much severe, than estimated from the phase margin of the open loop's Bode plot.
- Construction rules for the Bode plot:
 - a LHP pole adds -20dB/decade to the magnitude slope and -90° to the phase
 - a LHP (RHP) zero adds 20dB/decade to the magnitude slope and 90° (-90°) to the phase. The RHP zero adds overall 180° phase additionally.
 - a LHP conjugate complex pole adds a peak (height depending on Q) and -40dB/decade to the magnitude slope and -180° to the phase

Control theory basics III – Bode plot II



- The Bode plot shows, that with rising gain k_0 the stability worsens.
- What if the magnitude does not cross the 0dB point with -20dB/decade slope?
Solution: use the root locus plot (or numerically compute the closed loop TF / Bode plot).

Control theory basics III – Bode plot III

- For real poles and zeros the frequencies in the pole zero plot are identical of the kink frequencies in the Bode plot.
- Normalized PT₂ transfer function with conjugate complex poles: $G(s) = \frac{1}{\tau_2 \cdot s^2 + \tau_1 \cdot s + 1}$
- The unity gain frequency where -90° phase shift occurs is: $\omega_t = \frac{1}{\tau_2}$
- The damping coefficient is: $D = \frac{\tau_2}{2\tau_1}$
- Conjugate complex poles occur for $D < 1$. Peaking occurs for $D < \frac{\sqrt{2}}{2}$
- Real and imaginary parts of the poles are:

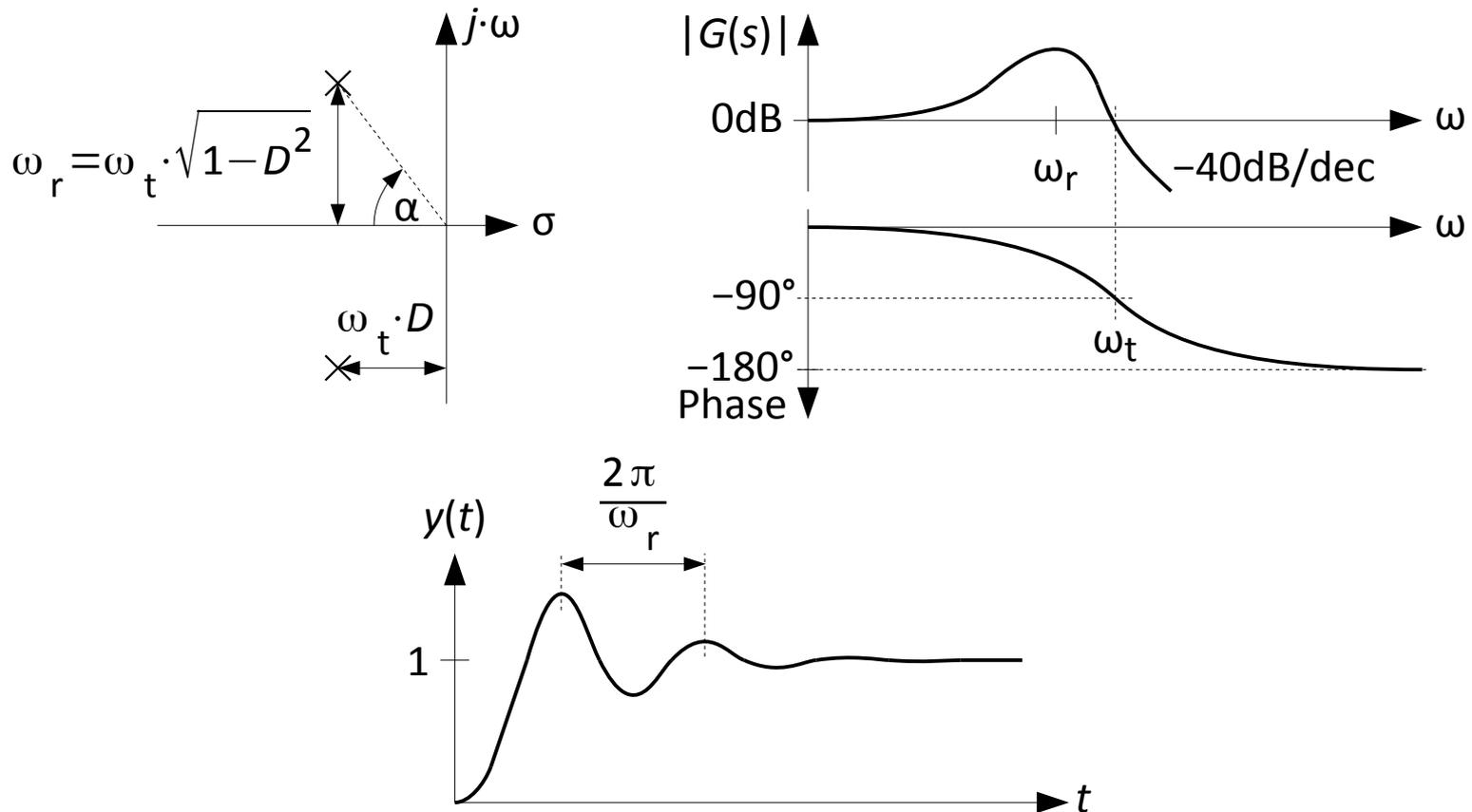
$$\Re(s_p) = -\omega_t \cdot D \quad \Im(s_p) = \pm \omega_t \cdot \sqrt{1 - D^2}$$
- Peaking frequency is:

$$\omega_r = \omega_t \cdot \sqrt{1 - D^2} = \Im(s_p)$$
- Ring frequency in time domain (step answer) is ω_r . The exponential decay constant is $\omega_t \cdot D$:

$$y(t) \propto e^{-\omega_t \cdot D \cdot t} \cdot \sin(\omega_r \cdot t + \varphi)$$

Control theory basics III – Bode plot IV

- Correspondence between pole zero plot (left), Bode plot (right) and transient step response (bottom). Figures for illustration only, not to scale.
- Damping coefficient D can be determined graphically: $D = \cos(\alpha)$

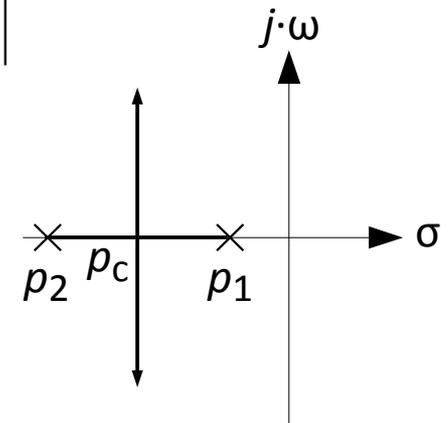


Control theory basics IV – Root locus plot I

- The root locus plot is a pole zero plot showing the *poles and zeros of the open loop*.
- Segments show the position of the *poles of the closed loop* as a function of k_0
- Elementary construction rules:
 - the root locus plot is symmetrical to the real (σ) axis,
 - the poles of the closed loop start at the poles of the open loop for $k_0=0$. n poles end at open loop zeros for $k_0 \rightarrow \infty$. $n-m$ poles end in infinity. The asymptotes are separated equally, that is with $360^\circ/(n-m)$ spacing.
 - a point on the real axis is part of the root locus, if there is an uneven number of total poles+zeros on the axis right from the point (don't count conjugate complex poles/zeros).
- Use a mathematical software tool like Matlab or Scilab to plot the root locus and get the frequency and critical gain values by clicking on the diagram.
 - For Scilab use the `evans()` and `krac2()` functions for root locus plots and critical gain determination, respectively.

Control theory basics IV – Root locus plot II

- 3 problematic cases: a) second order lowpass plant $A(s)$ with two real poles p_1 and p_2 and full feedback $B(s)=1$. As the open loop features real poles only, no ringing is expected. However, above a critical OpAmp DC gain k_c , the closed loop shows conjugate complex poles and ringing occurs for a step input.
 - The critical frequency p_c is given as: $p_c = -0.5 \cdot |p_1 + p_2|$
 - The critical gain k_c is given as: $k_c = \left| \left(p_c / p_1 - 1 \right) \cdot \left(p_c / p_2 - 1 \right) \right|$
 - Numerical example with $p_1 = -10\text{Hz}$, $p_2 = -10\text{MHz}$:
 $p_c \approx -5\text{MHz}$, $k_c \approx 2.5 \cdot 10^5 = 108\text{dB}$
 $Q = 0.7$ for $k_c = 114\text{dB}$
 - Numerical example with $p_1 = -10\text{Hz}$, $p_2 = -10\text{kHz}$:
 $p_c = -5005\text{Hz}$, $k_c = 249.5 = 48\text{dB}$
 $Q = 0.7$ for $k_c = 499.5 = 54\text{dB}$
 - Note: the closer the poles of the open loop to each other, the less gain is required to achieve conjugate complex poles for the closed loop.

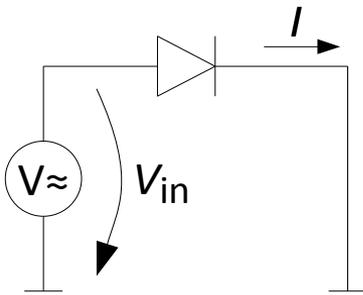


Practical issues I – What frequencies do I have? I

- The obvious ones: input/output signal frequencies, quartz & IC clock frequencies (don't forget neither the charge pump and switched-capacitor amplifier nor ADC/DAC clocks).
- The semi-obvious ones: signal/clock harmonics.
- The non-obvious ones I: nonlinear elements (e.g. diodes, transistors, varistors) generate intermodulation frequencies of clock (incl. harmonics) and I/O signal (incl. harmonics):
 - $f = a \cdot f_{\text{CLK}} + b \cdot f_{\text{signal}}$ with a, b integer numbers (may be negative, too). For the sum $|a| + |b|$ is called intermodulation order (usually only for $a \neq 0, b \neq 0$ to distinguish from harmonics).
 - Third order intermodulation frequencies (products) are:
 $\pm 2 \cdot f_{\text{CLK}} \pm 1 \cdot f_{\text{signal}}$ and $\pm 1 \cdot f_{\text{CLK}} \pm 2 \cdot f_{\text{signal}}$ (4 positive and 4 negative frequencies)
- The non-obvious ones II: broadband external noise (from neighbor PC, DC/DC converter, electrical machines, aerial broadcast, etc.) and local thermal noise.

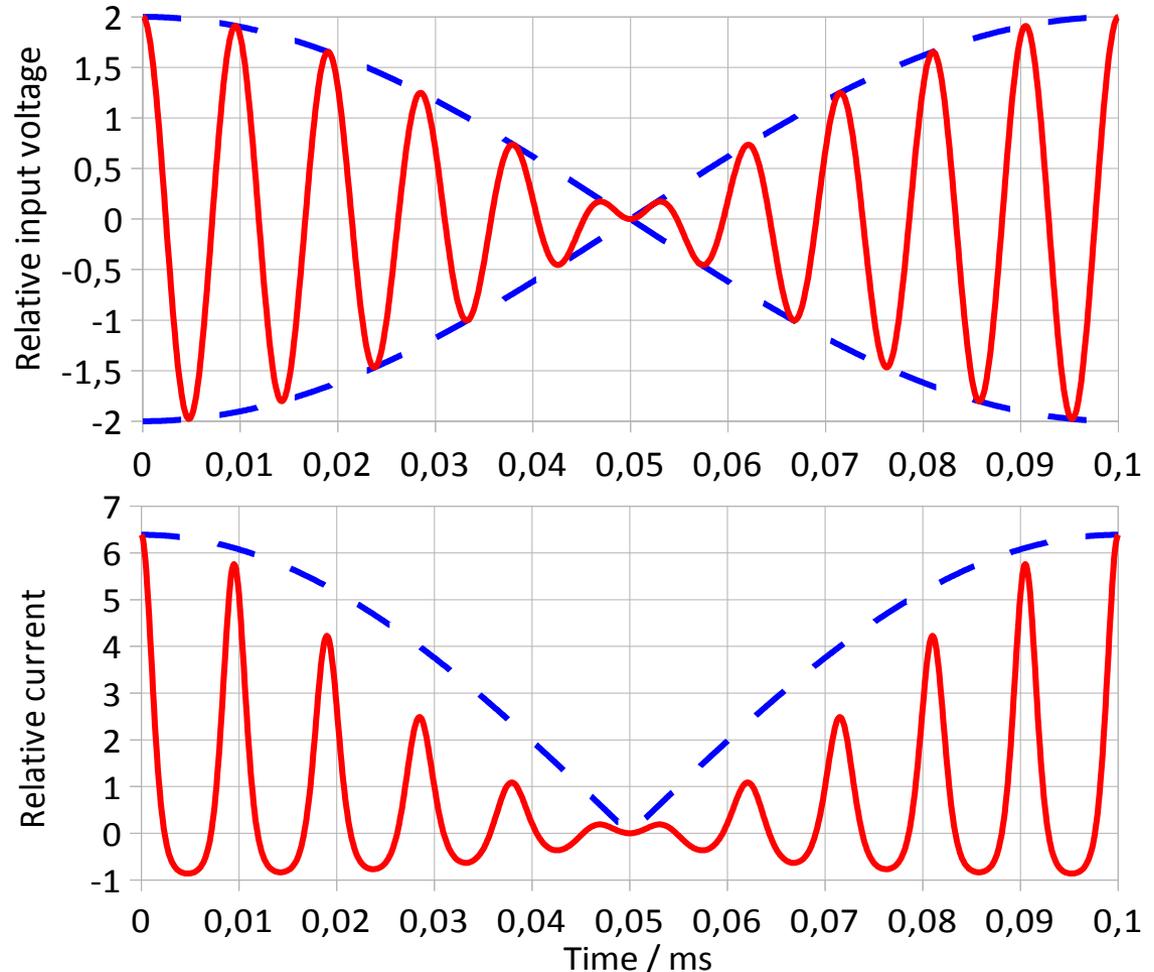
Practical issues I – What frequencies do I have? II

- Intermodulation at a diode. Two small sinusoidal input voltages with 100kHz and 110kHz. The current is rectified and the envelope is not sinusoidal. Nonlinearity increases with voltage amplitude.



$$V_{in}/V_T = \cos(100\text{kHz} \cdot 2\pi \cdot t) + \cos(110\text{kHz} \cdot 2\pi \cdot t)$$

$$I/I_0 = \exp(V_{in}/V_T) - 1$$



Practical issues I – What frequencies do I have? III

- The current spectrum is from DC to infinity with 110kHz–100kHz=10kHz spacing. The plot is normalized to have 0dB at the input frequencies of 100kHz and 110kHz, respectively.

