Inter-Surface Mapping

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| fd8 | fd8 | fd8 | fd8 |
| (a) Surface with edges from  (notice density of edges from left wing) | (b) Surface with edges from  (see spike flattened on rear left knee) | (c) normals mapped onto  (lit using 2 antipodal light sources) | (d) 50% morph |

Figure : Inter-surface map for two objects of genus 2, initialized with 8 user-specified feature points. (Symmetric stretch efficiency 0.311).

# Abstract

We consider the problem of creating a map between two arbitrary triangle meshes. Whereas previous approaches compose parametrizations over a simpler intermediate domain, we directly create and optimize a continuous map between the meshes. Map distortion is measured with a new symmetric metric, and is minimized during interleaved coarse-to-fine refinement of both meshes. By explicitly favoring low inter-surface distortion, we obtain maps that naturally align corresponding shape elements. Typically, the user need only specify a handful of feature correspondences for initial registration, and even these constraints can be removed during optimization. Our method robustly satisfies hard constraints if desired. Inter-surface mapping is shown using geometric and attribute morphs. Our general framework can also be applied to parametrize surfaces onto simplicial domains, such as coarse meshes (for semi-regular remeshing), and octahedron and toroidal domains (for geometry image remeshing). In these settings, we obtain better parametrizations than with previous specialized techniques, thanks to our fine-grain optimization.

**Keywords**: surface parametrization, shape morphing, remeshing.

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| *v*  ~  *u*  *w*  ~  ~  *M1* | | | *M2*  *v*  *u*  *w* | |
| (a) ; initial | | | (b) | |
| *v*  ^  *u*  *w*  ^  ^  *R2*  *I*  old *v*  ^ | | | *v*  ~  *u*  *w*  ~  ~  *M1* | |
| (c) | | | (d) ; new | |
| Figure : Illustration of neighborhoods in vertex optimization. | | | | |
| *v*  ^  *u*  *w*  ^  ^  *R2* | *v*  ~  ~  ~  *u*  *w*  *M1* | |

Figure . A kink vertex (red, right) is required since a direct segment along (dotted) goes on the wrong side of .

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| conformal |
| Figure . Use of a conformal metric results in a poor inter-surface map. |

# Applications and results

## Inter-surface mapping

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| teapot_mug copy | | | |
| Figure 6. Inter-surface map between two genus-1 objects. (a,b) use fixed constraints while (c,d) drop the constraints after initialization. (a,c) cup edges on teapot. (b) teapot edges on cup. (d) 50% morph. (Sym. stretch efficiencies: (a,b) 0.471, (c,d) 0.598). | | | |
| bound2 | bound2 | bound2 | eyedistortion2 |
|  | 50% morph |  | Close-up |
| Figure . Map between two meshes with boundaries. The close‑up on the eye shows low distortion around the feature point ( edges over geometry). (Symmetric stretch efficiency 0.967). | | | |

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| horse | cow | ch4 |
| horse base | cow base | 50% morph |

Figure : Cow-horse inter-surface map using only 4 features.

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| chremesh | ch4 |
| (a) composition of 2 simplicial maps | (b) direct inter-surface map |
| Figure . The inter-surface map automatically favors shape correspondence, unlike the composition of two separate simplicial parametrizations, as shown in these morphs. (The simplicial map uses the 17 feature points shown in Figure 2.) (Symmetric stretch efficiencies: (a) 0.416, (b) 0.442). | |

## Simplicial parametrization

In this scenario, is an abstract domain whose triangle faces are conceptually all equilateral. Although such a domain lacks an isometric embedding in , this is not a problem for the algorithm. During the construction of the local neighborhood in Section 5.1, the faces in are simply taken to be equilateral.

Among previous simplicial parametrization methods, the most advanced is the Globally Smooth Parametrization (GSP) work of Khodakovsky et al [2003], which attains smoothness across domain edges. However, it compresses the parametrization in the vicinity of low-valence irregular vertices, and stretches it near high-valence irregular vertices. As Figure 10 shows, our maps are visually smooth everywhere, and the extraordinary domain vertices have much less influence on the parametrization uniformity.

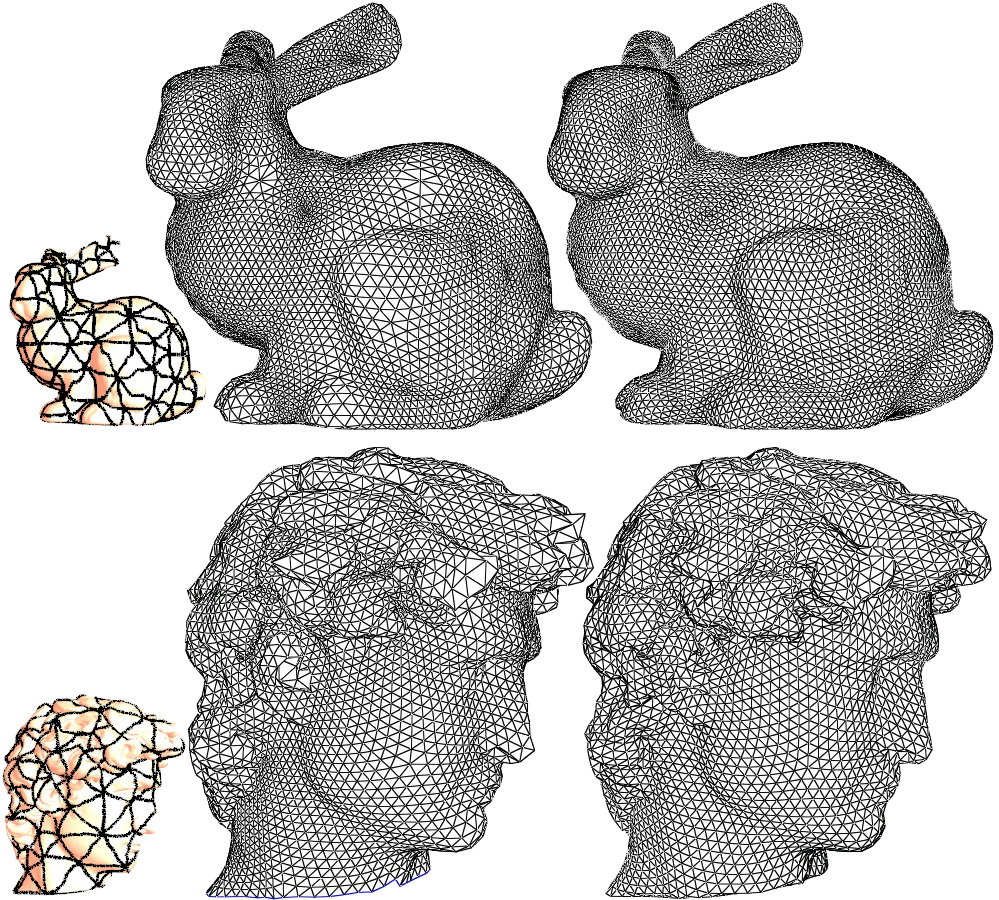
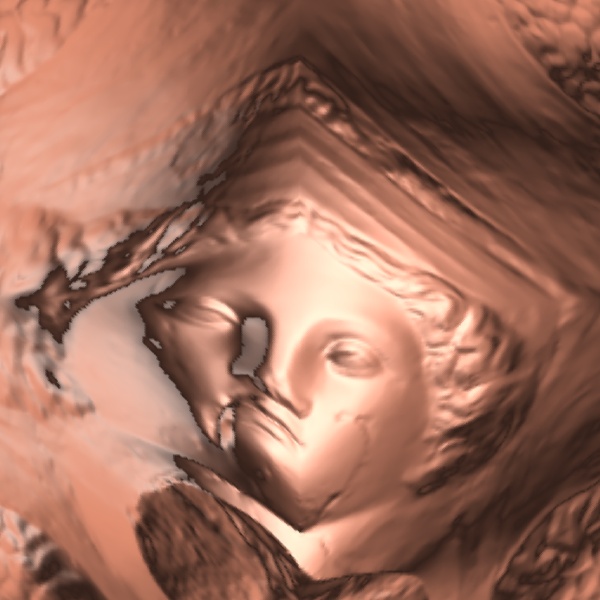


Figure . Comparison of semi-regular remeshing using GSP (middle) and our method (right), using the same set of base domain patches (left). (One-way stretch efficiencies: bunny 0.800, 0.915; David 0.761, 0.902).

## Octahedral parametrization

Praun and Hoppe [2003] use a sphere as an intermediate domain to parametrize a surface onto an octahedron, for subsequent geometry image remeshing. By directly optimizing the octahedron-to-surface map, we obtain improved results. The inset figure shows the Venus head as a geometry image obtained by unfolding an octahedral parametrization. As shown in Table 1, the parametrization stretch efficiency is improved in all cases, and the geometric accuracy of the remeshes (as measured with PSNR) is also improved for models with many extremities.

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| Model | One-way  stretch efficiency | | Remesh PSNR (dB) | |
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| Venus | 0.943 | 0.947 | 83.4 | 83.2 |
| Bunny | 0.706 | 0.717 | 80.0 | 79.9 |
| Gargoyle | 0.643 | 0.679 | 79.2 | 79.3 |
| Armadillo | 0.454 | 0.528 | 72.0 | 73.0 |
| Horse | 0.363 | 0.398 | 76.9 | 77.7 |
| Cow | 0.405 | 0.440 | 74.9 | 77.0 |
| tyrannosaurus | 0.360 | 0.418 | 73.6 | 74.5 |
| Table . Comparison of octahedral remeshing using spherical parametrization () [Praun and Hoppe 2003], and using our direct map onto octahedron domain . | | | | |

## Toroidal parametrization

A natural domain for genus-1 surfaces is the toroidal unit square. It is formed by identifying the square’s boundaries left-to-right and top-to-bottom. To apply our framework to this scenario, we let the toroidal domain be represented by a mesh with 9 vertices and 18 triangles. As in simplicial parametrization, the domain does not have a *global* isometric embedding in , but again we can use the local geometry of the domain when constructing the neighborhoods and in Section 5.1. In this case, the triangles in are always right isosceles triangles, and their configuration is such that is planar. Thus, the *local* map is always an isometry.

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| *Toroidal domain  tessellation* |

To initialize the parametrization, the user specifies 9 feature points on the input mesh , to correspond with the domain mesh vertices. To allow maximum freedom for the map, these feature points do not act as constraints during coarse-to-fine optimization. Figure 11 shows some example results.

There has been little work on toroidal parametrizations of arbitrary genus-1 surfaces, which is surprising since the domain is the most “Euclidean” of all closed surface topologies. Gu and Yau [2003] demonstrate their global conformal approach on genus-1 surfaces. Compared to their results, ours exhibit less scale-distortion due to the use of a stretch functional.

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| newteapot | newteapot |
| rockerarm | rockerarm |
| Surfaces mapped into toroidal domain (with 2-sided lighting) | Remeshed surfaces (all vertices have valence exactly 4) |
| Figure . Examples of toroidal parametrization and remeshing. (One-way stretch efficiencies: teapot 0.458, rocker arm 0.582). | |

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# Future work

There are several avenues for future work. To improve speed we envision using fine-to-coarse propagation of information [Sander et al 2002] to obtain better configurations at low resolutions. Huge meshes could be handled using a hybrid strategy; after running our ISM algorithm to create a good mid-resolution map, we could define the finer map using simplicial map composition, since the simplicial pieces may be small and flat enough to avoid numerical problems and geometric detail mismatch.

One exciting application is the use of inter-surface maps to automatically transfer geometric texture between models. This may allow surface texture synthesis using other surfaces as exemplars.

An interesting open question is how to extend our method to handle multiple models. Simultaneously optimizing an all-to-all map would not scale, while using one model as domain would lose some benefits of directly optimizing inter-surface maps.

Another area of future work is computing maps with singularities to allow correspondences between objects with different topologies. User input may be required to associate topological features and introduce singularities on some of the meshes.

# Acknowledgements

We thank Cyberware and the Digital Michelangelo Project at Stanford University for the 3D models, and Andrei Khodakovsky for sharing his Globally Smooth Parametrization data.

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