Chapter 1: Introduction

This Ph.D. thesis considers the problems of non-quadratic stability analysis and control design for continuous-time Takagi-Sugeno models. The goal is to develop new approaches to overcome the drawbacks of existing approaches in fuzzy control theory.

1.1. Background and Motivation

Physical systems are generally described by nonlinear models, which makes stability analysis a goal difficult to rich, classical approaches tend to approximate them by linear systems. However, the major drawback is that the linearized systems fail to completely represent the real plants that are highly nonlinear. Researchers have proposed several ways to deal with nonlinear systems; a linear parameter varying (LPV) presentation has been proposed by [Shamma, 1988] in order to approximate nonlinear systems, An LPV system is essentially a linear time-varying system which can be written in the form

$$\begin{cases} \dot{x} = A\big(\theta(t)\big)\,x + B\big(\theta(t)\big)\,u \\ y = C\big(\theta(t)\big)\,x + D\big(\theta(t)\big)\,u \end{cases}$$

Where θ is a time varying parameter vector. As such it has a structure which is similar to a linear time-invariant state space system, and control design methods with some similarity to linear state space methods can indeed be used. Although these models do not capture the nonlinear behavior of models [Bernal & Guerra , 2010].

Another alternative introduced by [Shamma & Cloutier, 1993] to write nonlinear systems of the form of quasi-LPV models, this representation is obtained through an exact transformation of the nonlinear states. A quasi-LPV system is de defined as a system where the state realization can be put in the following form:

$$\begin{cases} \dot{x}(t) = A\big(y(t)\big)\,x(t) + B\big(y(t)\big)\,u(t), \;\; x(0) = x_0 \\ y = Cx(t) \\ \forall t \geq 0,\; y(t) \in \Theta_y \subset \mathbb{R}^n \end{cases}$$

This class of models known also as Takagi-Sugeno models [Takagi & Sugeno , 1985] which consists in a set of linear models blended together with nonlinear functions called membership functions (MFs) which hold the convex-sum property [Tanaka & al, 2001]. It allows then to exactly represent a nonlinear model in a compact set of the state variables [Taniguchi & al, 2001],

T-S models may be extended to polynomial fuzzy models which consists in a convex sum of polynomials models. It has been recently proposed in [Tanaka & al, 2009] to represent efficiently a nonlinear system, especially when nonlinear terms are polynomials.
In this thesis, nonlinear systems represented in the form of both T-S and polynomial fuzzy models are considered.

1.2. Review of previous Works

Over the last three decades, the so-called Takagi-Sugeno models [Takagi & Sugeno, 1985] have reached a great attention in the control community. Since they allow a systematic stability analysis and controller design via linear matrix inequalities (LMIs)[Tanaka & wang, 2001] which can be efficiently solved by convex programming techniques already implemented in commercially available software [Boyd & al, 1994] The results for stability, stabilization, estimation [Tanaka & Wang, 2001], [Lendek & al, 2010], [Feng, 2006] are now converging towards quasi-LPV models results [Sherer & weiland, 2004].

T-S models are combined with different control laws, among which parallel distributed compensation (PDC) is considered a natural option since it is based on linear state feedbacks blended together using the same MFs of the T-S representation. Once a T-S model and a control law are proposed, the direct Lyapunov method is applied to obtain LMI conditions for stability analysis, control and observer design. [Tanaka & Wang , 2001], [Sala & al, 2005]. The stability of a T-S model is based on the Lyapunov theory, proving the existence of a common matrix $P>0$ such that $\dot{V}<0$, where $V(t)=x(t)^T Px(t)$ is a Lyapunov candidate function. Nonetheless, the quadratic approach presents serious limitations because its solutions are inherently pessimistic, i.e., there are stable or stabilizable models which do not have a quadratic solution [Sala & al, 2005], this conservativeness comes from different sources: the type of T-S model [Guerra & al, 2007], [Bouarar & al, 2010], the way the membership functions are dropped-off to obtain LMI expressions [Tuan & al, 2001][Sala & Arino, 2007], [Sala & Arino, 2007], the integration of membership-function information [Sala & Guerra, 2008], [Bernal & al, 2009], or the choice of Lyapunov function [Johanson & al, 1999], [tanaka & al, 2001], there was room for reducing this conservativeness by changing the choice of the Lyapunov function.
Researchers have proposed several Lyapunov functions to deal with these drawbacks:

In [Tanaka & al, 2003], [Blanco & al, 2001] Fuzzy Lyapunov functions (FLFs) were proposed, thus constituting the first non-quadratic framework for T-S models, Nevertheless, the time-derivative of the membership functions of the T-S model appears in the derivative of the Lyapunov function which make the resulting conditions non LMIs, for that several results propose just to bound them a priori [Bernal & al, 2006], [Mozelli & al, 2009]. This way of doing is not satisfactory because the verification of these bounds can only be done a posteriori on a case by case approach, especially when compared with the discrete-case [Guerra & Vermiren, 2004], [Ding & al, 2006], [Guerra & al, 2009]. Another drawback rises from the fact that authors bound the time-derivatives of the MFs assuming that they do not depend on the input, which turns out to be very restrictive. Moreover, the proposed control law makes use of the time-derivatives of the MFs through a classical PDC scheme, thus ignoring the non-quadratic nature of the involved Lyapunov function.

In [johansson & al, 1999], [Feng & al, 2004], [Feng & al, 2005] researchers proved that the use of piecewise Lyapunov functions (PWLFs) have effectively relaxed the referred pessimism, though they require the MFs to induce a polyhedral partition of the state space. Unfortunately, this condition on the MFs of those TS models obtained by sector nonlinearity approach is not fulfilled; moreover, the piecewise approach leads to bilinear matrix inequalities in the continuous-time context which cannot be optimally solved [Feng & al, 2005].

In [Rhee & Won, 2006], a line-integral Lyapunov function is proposed to circumvent the MFs' time-derivative obstacle, though the line integral is asked to be path-independent thus significantly reducing its applicability [Guelton & al, 2010].

All these approaches consider the problem of global stability which is far to be the general rule for nonlinear systems. Although they present some improvements which are particularly important and allows dealing with problems that were unfeasible before. A change of perspective for non-quadratic stability analysis of T-S models has been proposed in [Guerra & Bernal, 2009]. This approach employing a non-quadratic Lyapunov function (NQLF) and priori known bounds [Guerra & Bernal, 2009], [Bernal & Guerra, 2010], [Bernal & al, 2010] and [Guerra & al, 2011], reduces global goals to less exigent conditions, thereby showing that an estimation of the region of attraction can be found (local stability); this solution parallelizes nonlinear analysis and design for models that do not admit a global solution [Khalil, 2002].

1.3. Purposes and Contributions

The subject of this work is to develop new non-quadratic stability and stabilization conditions for continuous T-S fuzzy systems, based on non-quadratic Lyapunov functions, new non-quadratic stability conditions are derived in order to overcome the drawbacks of the quadratic approaches and the existing approaches.

A first motivation for the work of this thesis arises from the fact that most of stability conditions are based on quadratic Lyapunov functions which means that the aim can be reached by finding a common Lyapunov matrix $P > 0$ for all the sub-models. This renders stability results conservative and even a large number of systems can be stable without the existence of a quadratic Lyapunov function.

A second motivation is that in most of existing approaches dealing with stability and stabilization, the properties of the membership functions are not taking into account except the property of convexity. In other approaches, it is taking in consideration the upper bound for the time derivative of the premise membership function as assumed by [Tanaka & al. , 2001a], [Tanaka & al. , 2001b], [Tanaka & al. , 2001c], [Tanaka & al. , 2003].

A third motivation is that it has been shown that reducing global stability goals to something less restrictive will give a nice solution by providing an estimation of the stability domain (local asymptotic conditions), as it is usually the case for nonlinear models for which stability and/or stabilization cannot be reached globally.

The main contributions of this thesis are in both stability analysis and controller design:

The first contribution of this thesis is concerned with a relaxation in the latter sense which demands a change of perspective from global to local conditions. Non-quadratic Lyapunov functions has been proposed to analyze the stability of continuous time Takagi-Sugeno models which means that the objective can be reached after finding a number of $P_i > 0$.

The second contribution consists in a sum of squares (SOS) approach based first on polynomial fuzzy modeling providing a more effective representations of the nonlinear systems and second more relaxed stability conditions based on polynomial fuzzy Lyapunov function comparing to the LMI-Based approach. These SOS conditions can be solved numerically using the Matlab toolbox SOSTOOLS [Prajna & al, 2002].

The third contribution is the extension of the local results obtained for stability analysis to the control design of continuous time Takagi-Sugeno models, Based on non-PDC control law according to the non-quadratic nature of the Lyapunov function, new Local stabilization conditions have been obtained. The well-known problem of handling time-derivatives of membership functions (MFs) as to obtain conditions in the form of linear matrix inequalities (LMIs) is overcome by reducing global goals to the estimation of a region of attraction.

Another contribution in this thesis, A novel approach is proposed allowing the design of a robust local H_∞ controller for continuous time Takagi-Sugeno based on non-quadratic Lyapunov function, the method is based on a new form of non-PDC controller and by the mean of Finsler's Lemma, LMIs conditions can be obtained, the idea does not requires a bound for the input control, it only needs a priori bound of the states which is given from the domain of definition of the T-S models.

1.4. Chapters outline

This thesis is organized as follows:

Chapter 1 provides an introduction de the study.

Chapter 2 introduces Takagi-Sugeno models followed by the method used to the design of these models. A recall of the basic concepts and definitions of the theory of stability in the Lyapunov sense is given. Quadratic stability and stabilization conditions for continuous-time Takagi-Sugeno models are then presented. Semi definite programming techniques and a number of tools and properties are cited. The chapter finishes by a large discussion of the drawbacks of existing approaches trying to overcome the problems encountered when using classical approaches for stability and stabilization.

Chapter 3 is devoted to the first major contribution in this thesis, it presents new solutions for stability analysis problems for continuous time Takagi-Sugeno models. This chapter is based on a method first proposed by [Guerra & Bernal, 2009] allowing to obtain local results and better estimation of the region of attraction via non-quadratic Lyapunov functions. Some improvements are then given in order to obtain better relaxed stability conditions followed by illustrative examples to show the advantages of the proposed LMIs conditions. Moreover, we present polynomial fuzzy modeling and stability analysis, the stability conditions based on polynomial Lyapunov functions are represented in terms of SOS and can be numerically (partially symbolically) solved via the recently developed SOSTOOLS. To illustrate the validity and applicability of the proposed approach, a number of analysis and design examples are provided.

Chapter 4 is devoted to the second major contribution in this thesis, it extends the results obtained in chapter 3 for stability analysis to the control design. New non-quadratic approaches based on non-PDC controller and non-quadratic Lyapunov functions are proposed in order to obtain more relaxed results comparing with recent existing methods in non-quadratic control design and to prove stabilization of a large number of continuous-time Takagi-Sugeno models which do not admit a quadratic stabilization. Simulation results are then presented to show the effectiveness of the proposed approaches during this chapter.

Chapter 5 studies the design of a robust non-quadratic controller based on non-quadratic Lyapunov function, the goal in this chapter is to take into account during the controller design of the different perturbations and unknown inputs that can affect a nonlinear system, in order to obtain sufficient local conditions allowing to stabilize the proposed models with better attenuation of the external perturbations. In then, a robust H infinity controller is designed for the proposed model showing that the link between the controller gain and the Lyapunov function can be cut in a convenient manner via Finsler's lemma. Simulation examples are given to highlight the method's advantages.

Chapter 6 ends the thesis with some concluding remarks and recommendations for future work.

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