

MATHEMATICS

Single correct answer type:

1. If the maximum value of $\cos(\sin x)$ is a and the minimum value is b , then

- (A) $a = \cos b$
- (B) $a = b$
- (C) $b = \cos a$
- (D) $b = \sin a$

Solution: (C)

$$-1 \leq \sin x \leq 1 \Rightarrow \cos 1 \leq \cos(\sin x) \leq 1$$

$$a = 1 \text{ and } b = \cos 1 \Rightarrow b = \cos a$$

Ideal Time: 120

Difficulty level: 4

Skill: Application

Chapter Name: Trigonometric functions

Key Concept Name: Graphs of $\cos x$ and $\sin x$

2. If $\alpha, \beta \in R$, then the value of

$$|(4 - \sin^2 \alpha)_{\max} - (3 + \cos^2 \beta)_{\min}| \text{ is equal to}$$

- (A) 0
- (B) 1
- (C) 6
- (D) 7

Solution: (B)

$$0 \leq \sin^2 \alpha \leq 1 \Rightarrow -1 \leq -\sin^2 \alpha \leq 0 \Rightarrow 3 \leq 4 - \sin^2 \alpha \leq 4$$

$$0 \leq \cos^2 \beta \leq 1 \Rightarrow 3 \leq 3 + \cos^2 \beta \leq 4$$

$$\therefore |(4 - \sin^2 \alpha)_{\max} - (3 + \cos^2 \beta)_{\min}| = |4 - 3| = 1$$

Ideal Time: 70

Difficulty level: 3

Skill: Evaluation

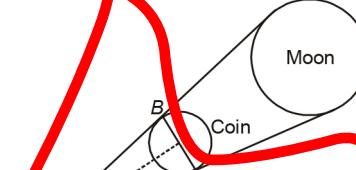
Chapter Name: Trigonometric functions

Key Concept Name: Range of $\sin x$ and $\cos x$

3. The approximate distance from the eye at which a coin of 2.2 cm diameter should be held so as to conceal the full moon with angular diameter 30, given that $(\pi = \frac{22}{7})$, is equal to

- (A) 15 cm
- (B) 152 cm
- (C) 25 cm
- (D) 252 cm

Solution: (D)



Let d be the required distance of a coin from the eye to conceal the full moon.

$$\begin{aligned}\theta &= 30' = \left(\frac{30}{60}\right)^\circ = \left(\frac{1}{2} \times \frac{\pi}{180}\right)^\circ \\ &= \left(\frac{\pi}{360}\right)^\circ\end{aligned}$$

Let arc AB = Diameter AB of coin

$$\begin{aligned}&= 2.2 \text{ cm} \\ \therefore d &= \frac{s}{\theta} = \frac{2.2 \times 360}{\pi} \text{ cm} \\ &= 252 \text{ cm}\end{aligned}$$

Ideal Time: 75

Difficulty level: 3

Skill: Evaluation

Chapter Name: Trigonometric functions

Key Concept Name: Basics of angles

4. If $\sec x + \sec^2 x = 1$, then value of $\tan^8 x - \tan^4 x - 2\tan^2 x =$

- (A) 1
- (B) -1
- (C) 0
- (D) 2

Solution: (A)

$$\sec x = -\sec^2 x + 1 = -\tan^2 x \Rightarrow \sec^2 x = \tan^4 x$$

$$1 + \tan^2 x = \tan^4 x$$

$$1 + 2\tan^2 x + \tan^4 x = \tan^8 x$$

$$\Rightarrow \tan^8 x - \tan^4 x - 2\tan^2 x = 1$$

Ideal Time: 120

Difficulty level: 4

Skill: Evaluation

Chapter Name: Trigonometric functions

Key Concept Name: Trigonometric identity

$$\Rightarrow \frac{2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)}{2 \sin \frac{C}{2} \cdot \cos \frac{C}{2}} = \frac{\cos \frac{C}{2}}{\sin \frac{C}{2}}$$

$$\Rightarrow \cos\left(\frac{A-B}{2}\right) = \cos \frac{C}{2} \Rightarrow A-B = C$$

$$A = B + C$$

$$\because A + B + C = \pi \Rightarrow 2A = 180^\circ \Rightarrow A = 90^\circ$$

Ideal Time: 150

Difficulty level: 6

Skill: Evaluation

Chapter Name: Trigonometric functions

Key Concept Name: Properties of triangle

10. If solution of $12 \cos^3 x - 3 \cos y = a$ lies on the line $y = 3x$, then exhaustive range of a is
- (A) $(-9, 9)$ (B) $[-9, 9]$
 (C) $(-\infty, -9) \cup [9, \infty]$ (D) $[-3, 3]$

Solution: (B)

$$12\cos^3 x - 3\cos 3x = a$$

$$\Rightarrow 12\cos^3 x - 3[4\cos^3 x - 3\cos x] = a$$

$$\Rightarrow 12\cos^3 x - 12\cos^3 x + 9\cos x = a$$

$$\Rightarrow 9\cos x = a \Rightarrow a \in [-9, 9]$$

Ideal Time: 120

Difficulty level: 5

Skill: Evaluation

Chapter Name: Trigonometric functions

Key Concept Name: Multiple angle formula

11. In a triangle ABC , $b = 3$, $c = 4$ and $\angle A = 90^\circ$ then the length of internal angle bisector of $\angle A$ is

$$\begin{array}{ll} \text{(A)} \frac{12\sqrt{2}}{7} & \text{(B)} \frac{6}{7\sqrt{2}} \\ \text{(C)} \frac{6\sqrt{2}}{7} & \text{(D)} \frac{6}{7} \end{array}$$

Solution: (A)

$$\text{Length of angle bisector of } \angle A = \frac{2bc}{b+c} \cos \frac{A}{2}$$

$$= \frac{2 \times 3 \times 4}{3+4} \cdot \cos 45^\circ = \frac{12 \times 2}{7} \times \frac{1}{\sqrt{2}} = \frac{12\sqrt{2}}{7}$$

Ideal Time: 70

Difficulty level: 3

Skill: Rote Learning

Chapter Name: Trigonometric functions

Key Concept Name: Geometry

12. The value of $\cos \frac{\pi}{11} + \cos \frac{3\pi}{11} + \cos \frac{5\pi}{11} + \cos \frac{7\pi}{11} + \cos \frac{9\pi}{11}$ is
- (A) 0 (B) $\frac{1}{2}$
 (C) 1 (D) 2

Solution: (B)

Use $\cos \alpha + \cos(\alpha + \beta) + \cos(\alpha + 2\beta) + \dots + \cos(\alpha + (n-1)\beta)$

$$= \frac{\cos\left(\alpha + \frac{(n-1)\beta}{2}\right) \frac{\sin n\beta}{2}}{\sin \frac{\beta}{2}}$$

$$\cos \frac{\pi}{11} + \cos \frac{3\pi}{11} + \cos \frac{5\pi}{11} + \cos \frac{7\pi}{11} + \cos \frac{9\pi}{11}$$

$$= \frac{\cos\left(\frac{\pi}{11} + \frac{4\pi}{11}\right) \sin \frac{5\pi}{11}}{\sin \frac{\pi}{11}} = \frac{1}{2}$$

Ideal Time: 150

Difficulty level: 6

Skill: Evaluation

Chapter Name: Trigonometric functions

Key Concept Name: Transformation formula

13. The value of $\cot \theta - \tan \theta - 2\tan 2\theta - 4\tan 4\theta - 8\cot 8\theta$, is
- (A) -1 (B) 0
 (C) 1 (D) 2

Solution: (B)

Ideal Time: 120

Difficulty level: 5

Skill: Evaluation

Chapter Name: Trigonometric functions

Key Concept Name: Transformation formula

14. The maximum value of $12\sin \theta - 9\sin^2 \theta$ is
- (A) 3 (B) 4
 (C) 5 (D) 6

Solution: (B)

Ideal Time: 70

Difficulty level: 3

Skill: Evaluation

Chapter Name: Trigonometric functions

Key Concept Name: Range of trigonometric functions

15. If $\cos(A + B) = \frac{4}{5}$ and $\tan A \cdot \tan B = \frac{1}{3}$, then $\sin A \sin B$, is

(A) $\frac{3}{5}$	(B) $\frac{2}{5}$
(C) $\frac{4}{5}$	(D) $\frac{1}{5}$

Solution: (B)

$$\cos(A + B) = \frac{4}{5}$$

$$\tan A \tan B = \frac{1}{3}$$

$$\Rightarrow 3\sin A \sin B = \cos A \cos B$$

$$\Rightarrow \cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\frac{4}{5} = 3\sin A \sin B - \sin A \sin B$$

$$\Rightarrow 2 \sin A \sin B = \frac{4}{5} \Rightarrow \sin A \sin B = \frac{2}{5}$$

Ideal Time: 120

Difficulty level: 5

Skill: Evaluation

Chapter Name: Trigonometric functions

Key Concept Name: Transformation formula

16. The value of $\cos 75^\circ - \sin 105^\circ$ is

(A) $-\frac{1}{2\sqrt{2}}$	(B) $-\frac{1}{\sqrt{2}}$
(C) $\frac{1}{\sqrt{2}}$	(D) $\frac{1}{2\sqrt{2}}$

Solution: (B)

$$\cos 75^\circ - \sin 105^\circ = \sin 15^\circ - \sin 75^\circ$$

$$= \left(\frac{\sqrt{3}-1}{2\sqrt{2}} \right) - \left(\frac{\sqrt{3}+1}{2\sqrt{2}} \right)$$

$$= -\frac{1}{\sqrt{2}}$$

Ideal Time: 75

Difficulty level: 3

Skill: Evaluation

Chapter Name: Trigonometric functions

Key Concept Name: Transformation formula

17. The value of $\frac{\sin 2\alpha - \sin 3\alpha + \sin 4\alpha}{\cos 2\alpha - \cos 3\alpha + \cos 4\alpha}$ is

(A) $\tan 2\alpha$	(B) $\cot 2\alpha$
(C) $\tan 3\alpha$	(D) $\cot 3\alpha$

Solution: (C)

$$\begin{aligned} \frac{\sin 2\alpha - \sin 3\alpha + \sin 4\alpha}{\cos 2\alpha - \cos 3\alpha + \cos 4\alpha} &= \frac{2 \sin 3\alpha \cdot \cos \alpha - \sin 3\alpha}{2 \cos 3\alpha \cdot \cos \alpha - \cos 3\alpha} \\ &= \frac{\sin 3\alpha}{\cos 3\alpha} \end{aligned}$$

$$= \tan 3\pi$$

Ideal Time: 75

Difficulty level: 3

Skill: Evaluation

Chapter Name: Trigonometric functions

Key Concept Name: Transformation formula

18. The value of expression

$$\frac{2(\sin 1^\circ + \sin 2^\circ + \sin 3^\circ + \dots + \sin 89^\circ)}{2(\cos 1^\circ + \cos 2^\circ + \cos 3^\circ + \dots + \cos 44^\circ) + 1}$$

equals

(A) $\sqrt{2}$	(B) $\frac{1}{\sqrt{2}}$
(C) $\frac{1}{2}$	(D) 0

Solution: (A)

$$\frac{2 \{ (\sin 1^\circ + \sin 89^\circ) + (\sin 2^\circ + \sin 88^\circ) + \dots + (\sin 44^\circ + \sin 46^\circ) + \sin 45^\circ \}}{2(\cos 1^\circ + \cos 2^\circ + \cos 3^\circ + \dots + \cos 44^\circ) + 1}$$

$$= \frac{2 \{ 2 \sin 45^\circ (\cos 44^\circ + \cos 43^\circ + \dots + \cos 1^\circ) + \sin 45^\circ \}}{2(\cos 1^\circ + \cos 2^\circ + \cos 3^\circ + \dots + \cos 44^\circ) + 1}$$

$$= \frac{2 \sin 45^\circ \{ 2(\cos 1^\circ + \cos 2^\circ + \dots + \cos 44^\circ) + 1 \}}{2(\cos 1^\circ + \cos 2^\circ + \dots + \cos 44^\circ) + 1}$$

$$= 2 \cdot \frac{1}{\sqrt{2}}$$

$$= \sqrt{2}$$

(4)

Ideal Time: 120

Difficulty level: 5

Skill: Evaluation

Chapter Name: Trigonometric functions

Key Concept Name: Transformation formula

19. If a, b, c represent the sides of a $\triangle ABC$ such that $a^2 + b^2 + c^2 - 2(a + b + c) + 3 = 0$, then area of $\triangle ABC$ is

- (A) $\frac{\sqrt{3}}{4}$ sq. units (B) 3 sq. units
 (C) $\sqrt{3}$ sq. units (D) 4 sq. units

Solution: (A)

$$\therefore a^2 + b^2 + c^2 - 2(a + b + c) + 3 = 0$$

$$\therefore (a - 1)^2 + (b - 1)^2 + (c - 1)^2 = 0$$

$$\therefore a = b = c$$

$$\therefore \text{Area of } \triangle ABC = \frac{\sqrt{3}}{4} a^2 = \frac{\sqrt{3}}{4} \text{ sq. units}$$

Ideal Time: 120

Difficulty level: 5

Skill: Application

Chapter Name: Trigonometric functions

Key Concept Name: Properties of triangle

20. If A, B and C are angles of a triangle, then $\frac{\sin 2A + \sin 2B + \sin 2C}{\sin A + \sin B + \sin C} =$

- (A) $8 \sin \frac{A}{2} \cdot \cos \frac{B}{2} \cdot \cos \frac{C}{2}$
 (B) $8 \sin \frac{A}{2} \cdot \sin \frac{B}{2} \cdot \sin \frac{C}{2}$
 (C) $8 \cos \frac{A}{2} \cdot \cos \frac{B}{2} \cdot \sin \frac{C}{2}$
 (D) $8 \cos \frac{A}{2} \cdot \cos \frac{B}{2} \cdot \cos \frac{C}{2}$

Solution: (B)

$$\therefore \sin 2A + \sin 2B + \sin 2C = 4 \sin A \times \sin B \times \sin C$$

$$\text{and } \sin A + \sin B + \sin C = 4 \cos \frac{A}{2} \cdot \cos \frac{B}{2} \cdot \cos \frac{C}{2}$$

$$\text{and } \sin A = 2 \sin \frac{A}{2} \cdot \cos \frac{A}{2}$$

Ideal Time: 75

Difficulty level: 3

Skill: Rote Learning

Chapter Name: Trigonometric functions

Key Concept Name: Properties of triangle

21. The values of θ in interval $(0, 2\pi)$ which satisfy the equation $\tan 2\theta \cdot \tan \theta = 1$ is

- (A) $\frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}$
 (B) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$
 (C) $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$
 (D) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$

Solution: (B)

$$\therefore \tan 2\theta \cdot \tan \theta = 1$$

$$\therefore \tan 2\theta = \cot \theta$$

$$\tan 2\theta = \tan (\frac{\pi}{2} - \theta)$$

$$\therefore x = \frac{(2n+1)\pi}{6}, n \in I, \text{ But } x \neq 1, 4 \dots$$

Ideal Time: 120

Difficulty level: 4

Skill: Evaluation

Chapter Name: Trigonometric functions

Key Concept Name: General solutions

22. If the sides of a triangle are in arithmetical progression then

- (A) $\cot \frac{A}{2}, \cot \frac{B}{2}, \cot \frac{C}{2}$ are in Arithmetico- Geometric progression
 (B) $\cot \frac{A}{2}, \cot \frac{B}{2}, \cot \frac{C}{2}$ are in G.P.
 (C) $\tan \frac{A}{2}, \tan \frac{B}{2}, \tan \frac{C}{2}$ are in A.P.
 (D) $\tan \frac{A}{2}, \tan \frac{B}{2}, \tan \frac{C}{2}$ are in H.P.

Solution: (D)

a, b, c are in AP.

$s-a, s-b, s-c$ are in AP

$\frac{r}{s-a}, \frac{r}{s-b}, \frac{r}{s-c}$ are in HP

$\tan \frac{A}{2}, \tan \frac{B}{2}, \tan \frac{C}{2}$ are in HP

Ideal Time: 150

Difficulty level: 6

Skill: Application

Chapter Name: Trigonometric functions

Key Concept Name: Properties of triangle

