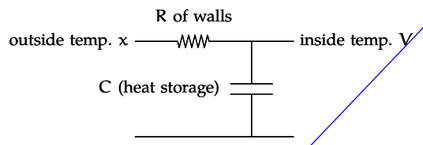
6 Simulating daily temperature variations in a house

In this problem, you model the temperature in a house as a function of time of day. A crude but very useful model is as a leaky tank or RC circuit. The resistor is the thermal resistance of the walls. The capacitor is the heat-storage-capacity of the house (mostly in the walls and floor). A typical house may have a (thermal) time constant of $\tau = 6$ hours.

The leaky-tank or RC-circuit difference equation is

$$V[n] = (1 - \alpha)V[n-1] + \alpha x[n], \tag{24}$$

where $\alpha \equiv T/\tau$ is the dimensionless time step, and $x[n]$ is the input signal–perhaps a temperature or a voltage.



In Boston during late spring, the outside temperature x is, say,

$$x(t) = 55\,^\circ\text{F} + 15\,^\circ\text{F} \times \cos\frac{2\pi t}{24\,\text{hours}}, \tag{25}$$

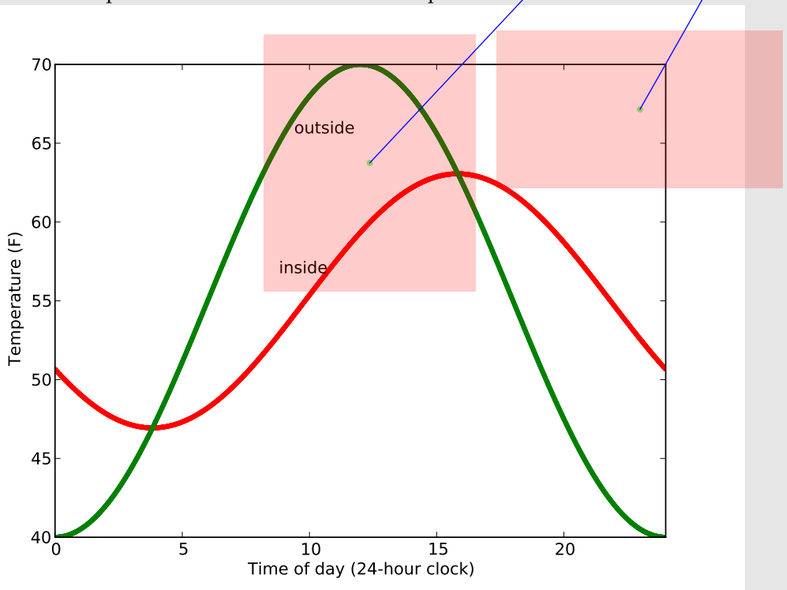
where $t = 0$ is noon on, say, May 1.

a. Sketch $x(t)$.

I sketched x and V on the same axes (see the next part).

b. Simulate the leaky-tank equation to find and plot the output signal V—namely, the temperature in the house—choosing a reasonable time step T (and therefore α).

Here is a plot of the inside and outside temperatures:



Comments on page 6

Were we supposed to plot both signals on the same graph?

My simulation was slightly different at the beginning when both, the house and outside temperatures, started at the same point. However, then it settled down to look very similar to this.

My graphs started at 12, noon, so they both started at the same value of 70 deg F. I think the period equals out in the end. I should examine this.