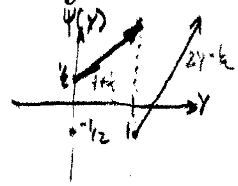
MAE 281B: Nonlinear Control Systems.



May 3, 2007

1. (2 points) Determine the describing function of the nonlinearity

$$\psi(y) = \begin{cases} y + \frac{1}{2}, & \text{for } 0 \le y \le 1\,, \\ 2y - \frac{1}{2}, & \text{for } y \ge 1\,. \end{cases}$$

$\Psi(a) = \frac{2}{\pi a}\int (a\sin\theta)\sin\theta\, d\theta + \frac{2}{\pi a}\int \frac{1}{2}\sin\theta$

2. (2 points) A servo motor is controlled with a relay histeresis, see Figure 1. Predict the frequency and amplitude of possible limit cycles of the system. The describing function of the relay satisfies:

$$\frac{-1}{\Psi(A)} = \frac{-\pi A}{4}\sqrt{1 - \frac{1}{4A^2}} - j\frac{\pi}{8}\,.$$

DESCRIBING FUNCTION

FREQ + AMP OF LIMIT CYCLE

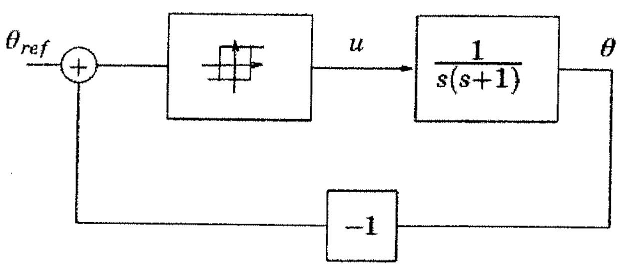
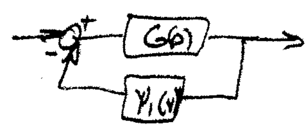


Figure 1: Diagram for Problem 2



3. (2 points) A stable linear system $G(p)$ and a nonlinear function $\psi_1(y) = y + \arctan(y)$ are connected under negative feedback. What conditions must be put on the linear part $G(p)$ for stability to be guaranteed by the circle criterion? HURWITZ FUNCTION $G(P)$

4. (2 points) Show that the system $\ddot{y} - (1 - y^2)\dot{y} + y = 0$ does not have a limit cycle in the region $|y| < 1$. BENDIXSON

$\frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial x_2} \neq 0$

5. (2 points) A motor with a time-varying dynamics is controlled with a simple proportional controller. An equivalent representation of the resulting closed-loop system is the feedback interconnection of $G(s) = \frac{1}{(s+1)^2}$ with the map $\varphi(t, y) = a\sin(\omega t)y$. Find a sufficient condition on a so that the closed loop system is finite-gain $\mathcal{L}_\infty$ stable. *Hint:* You may want to use the the convolution formula $y(t) = \int_0^t h(t - \sigma)u(\sigma)d\sigma$, where $h(t) = \mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2}\right\} = te^{-t}$, to find the $\mathcal{L}_\infty$ gain of $G(s)$.

$\dot{V}$ $\|x\|^2 + \|x\|\,|u|$