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Classical dynamical polarization effects due to Coulomb potential between deformed nuclei

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The classical dynamics of nuclear polarization effects are studied at sub-Coulomb energies to estimate the orientation probability of deformed nuclei at the classical turning point. Implications of this study for the formation of giant nuclear molecules and for sub-Coulomb transfer of one neutron in $^{238}U+^{238}U$ collisions are pointed out.

The lowering of the barrier height and the possible minima or "pockets" in the interaction potential due to the nuclear orientation effects of the colliding heavy ions have been studied by many authors.[1] The most "favorable" configuration is found to be the one with their "ends touching," colloquially called the "nose-to-nose" configuration. The interest in these studies of pockets in heavy-ion potentials increased very much after Greiner and his collaborators[2] suggested the possibility of forming a giant nuclear molecule in a $^{238}U+^{238}U$ collision, like the ones observed in light nuclei.[3] Using the interaction potential calculated in a double-folding model, Oberacker[4] has recently estimated that ^{238}U nuclei at $E_{\rm lab}=6$ MeV/nucleon, forming a long-lived dinuclear system, must approach with orientation angles around the nose-to-nose configuration. The polarization effects of the colliding nuclei are also shown[5] to be important for the sub-Coulomb transfer of one neutron in central and near central collisions of ^{238}U on ^{238}U. Depending on whether one uses the center-to-center or the surface-to-surface distance of closest approach, the favored configurations are, respectively, the belly-to-belly and nose-to-nose configurations.

An apparent question of relevance to the abovementioned study is the probability of occurrence of the favorable alignments on the dynamical path during the collision if the nuclei are initially unpolarized and thus approach each other with equal probability for various orientations. Oberacker[4] showed by a semiclassical calculation of the Coulomb excitation in a $^{238}U+^{238}U$ collision that the probability for a favorable nose-to-nose alignment due to the dynamical orientation of the nuclear quadrupole moments (retaining only the monopole-multipole terms in the multipole expansion of the potential) is only about 1%. Integrating over the angular cones of 0°–35° and 145°–180°, the total probability of aligning one ^{238}U nucleus favorably at 6 MeV/nucleon is only about 10%. In this paper, we present a classical, dynamical calculation for the excitation of two deformed ^{238}U nuclei in their Coulomb field and compare the results with those of Oberacker.[4] The favorable alignment for each nucleus in a $^{238}U+^{238}U$ collision is found to be about 30% larger than that predicted by Oberacker.[4]

Our classical dynamical model consists of solving the Hamiltonian equations of motion

$$\dot{q}_\nu=\frac{\partial H}{\partial p_\nu},\ \dot{p}_\nu=-\frac{\partial H}{\partial q_\nu},\ \nu=1,2,\ldots,13\ , \tag{1}$$

with the Hamiltonian

$$H=T(p,q)+V(q)\ . \tag{2}$$

Here, the collective coordinates q_ν (and their canonically conjugate momenta p_ν) are the relative vector $\mathbf{R}=(R,\theta,\phi)$ between the nuclear centers of mass, the Euler angles $\Omega_k=(\phi_k,\theta_k,\psi_k)$ defining the orientation of the intrinsic principal axes of the two nuclei ($k=1,2$) with respect to the laboratory system, and the coordinates for the intrinsic quadrupole deformations β_k and γ_k. Since we are concerned here with energies below and at the most up to the barrier, the potential V in (2) consists of the Coulomb interaction and the deformation energy

$$V=V_c(q_\nu)+\frac{1}{2}\sum_{k=1}^{2}[C_{\beta k}(\beta_k-\beta_{0k})^2+C_{\gamma k}\gamma_k^2]\ , \tag{3}$$

where $V_c(q)$ is the Coulomb potential between the two deformed and oriented nuclei, for which we use the expression (A.17) of Ref. 6, written for two nonoverlapping charge distributions up to quadratic terms in the deformation coordinates. The stiffness parameters $C_{\beta k}$ and $C_{\gamma k}$ and the mass parameters defining the kinetic energy T of rotation and vibration of the nuclei in (2) are taken from the rotation-vibration model. We assume $\gamma_k=0$ in the following.

Because of the deformations of the nuclei, the trajectory is no more a Rutherford trajectory, though the changes are rather small. This is illustrated in Fig. 1(a) for $^{238}U+^{238}U$ at three different energies by taking the initial configuration in a plane with orientations $\theta_1=45°$ and $\theta_2=0°$ (shown in the figure). In this figure are also shown the variations of the deformations β_1 and β_2 and the angles of orientations θ_1 and θ_2 with time [Figs. 1(b) and 1(c), respectively]. We notice that during the approach (small times) both the deformations and orientations do not change much from their initial values. However, as the two nuclei reach the turning point at $R=R_{\rm min}$, larger changes occur in these parameters. At the distance of closest approach, $R=R_{\rm min}$, the values of deformations $\beta_{k=1,2}$ and angles of orientations $\theta_{k=1,2}$ depend very strongly on the initial configurations. This is illustrated in Fig. 2, where we show the orientation angles $\theta_1(R_{\rm min})=\theta_2(R_{\rm min})=\theta_c$ and the deformation parameters $\beta_1(R_{\rm min})=\beta_2(R_{\rm min})=\beta_c$ at the distance of closest approach as functions of the initial orientation angles $\theta_1=\theta_2=\theta_i(\beta_1=\beta_2=\beta_i=0.26)$. (For equal initial orientations and deformations of the nuclei, the changes in

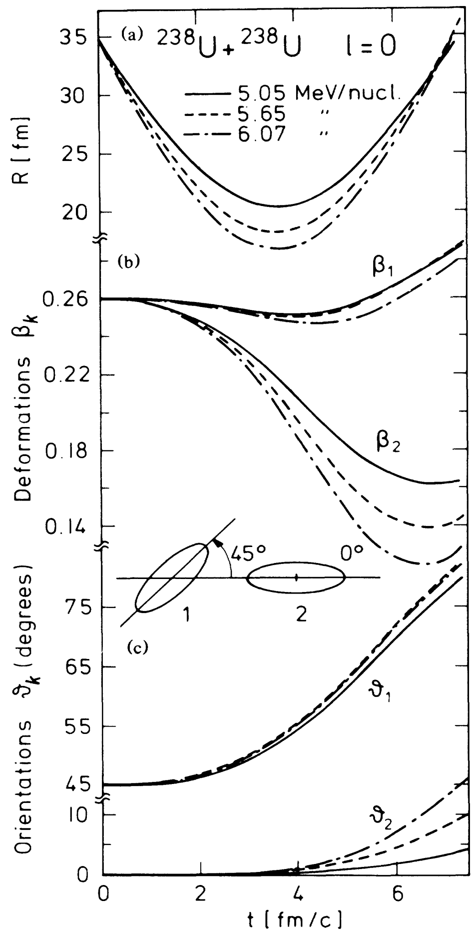

FIG. 1. (a) The classical Coulomb trajectories for central collisions of ^{238}U on ^{238}U at three different incident energies below the barrier, having an initial configuration with orientations $\theta_1=45°, \theta_2=0°$ in a plane. In (b) and (c) are shown, respectively, the variations of deformations β_1 and β_2 and the orientations θ_1 and θ_2 as functions of the collision time t. Initially we set $\beta_1=\beta_2=0.26$, $\theta_1=45°$, and $\theta_2=0°$.

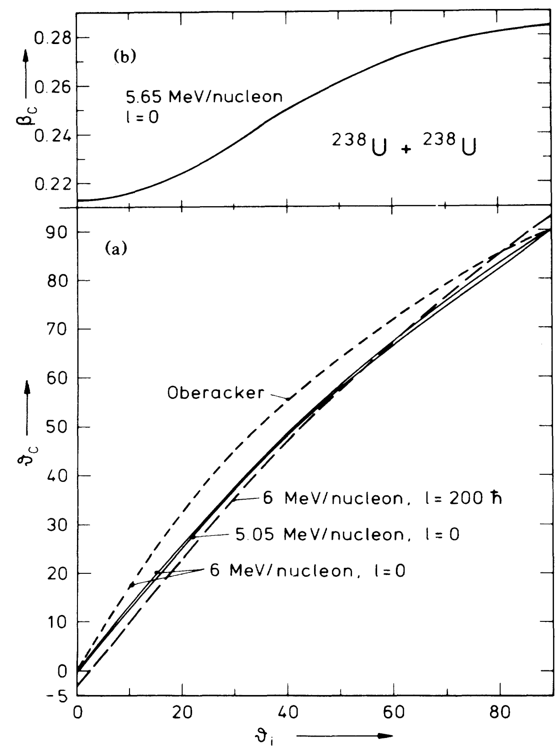

FIG. 2. (a) The initial orientations $\theta_i(=\theta_1=\theta_2)$ vs the orientations θ_c at the classical turning point for ^{238}U on ^{238}U at different energies and angular momenta. The short-dashed curve is Oberacker's result by using Eqs. (4) and (6). (b) The deformations β_c for equal initial deformations $\beta_i(=\beta_1=\beta_2)=0.26$ at the classical turning point, as function of the initial orientation θ_i for ^{238}U on ^{238}U at 5.65 MeV/nucleon and $l=0$.

these parameters during the collision are of equal amounts.) Almost independently of the incident energy (5.05 MeV/nucleon $\leqq E_{\text{lab}} \leqq 6.07$ MeV/nucleon) and orbital angular momentum ($l \leqq 200\hbar$), large changes in angles of orientations, $\Delta\theta=8°$, occur when the nuclei are oriented at 45° (or 135°). Similar results hold good for configurations with unequal initial orientation angles. In cases of nose-to-nose and belly-to-belly configurations, for the central collisions ($l=0$) there is no change in orientation angles, and at finite impact parameters only small changes occur ($\Delta\theta=-3°$ for $l=200\hbar$). On the other hand, Fig. 2(b) shows that the change in deformations is largest for the nose-to-nose and belly-to-belly configurations and almost zero for $\theta_1=\theta_2=\theta_i=45°$ (or 135°).

In Fig. 2(a), we have also compared our results of classical trajectory calculations with those of Oberacker[4] who carried out a semiquantal treatment of inelastic Coulomb excitation of rotational states at 6 MeV/nucleon for central collisions. In close proximity with the calculations of Oberacker, we assume that the probability of orientation of *each* nucleus $k=1,2$ having an initial orientation θ_k^i is given by $dP_k=0.5\sin\theta_k^i d\theta_k^i$. Then, parametrizing our calculated curve for 6 MeV and $l=0$, shown in Fig. 2(a), by (angles in radians, $\theta_1^i=\theta_2^i=\theta_i$)

$$\theta_i=\theta_c+b\theta_c\left(\frac{\pi}{2}-\theta_c\right)+c\theta_c\left(\frac{\pi}{2}-\theta_c\right)^2 , \tag{4}$$

we can write the orientation probability in terms of the orientation angles θ_c at the distance of closest approach $R_{\min}$. We get

$$dP=0.5\sin\theta_i\left[1+b\frac{\pi}{2}-2b\theta_c+c\left(\frac{\pi}{2}-\theta_c\right)^2 -2c\theta_c\left(\frac{\pi}{2}-\theta_c\right)\right]d\theta_c . \tag{5}$$

Oberacker has plotted $dP/(\sin\theta_c d\theta_c)$ as a function of θ_c at the distance of closest approach $R_{\min}=17$ fm, taken fixed (actually the distance of closest approach changes with the initial orientation of nuclei). Then the constants

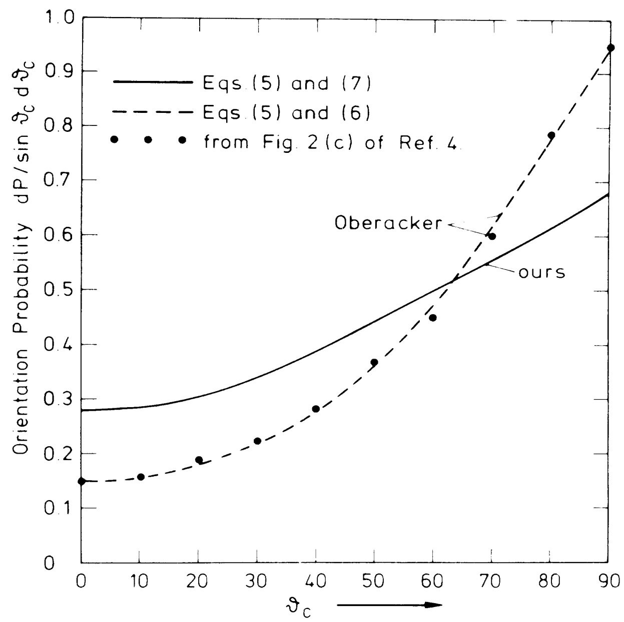

FIG. 3. Orientation probability $dP/(\sin\theta_c d\theta_c)$ vs the angle of orientation θ_c at the classical turning point for ^{238}U on ^{238}U at 6 MeV/nucleon and $l=0$.

b and c, which give the measure of the change in orientation ($\Delta\theta=\theta_c-\theta_i$), are determined by using Oberacker's[4] values of $dP/(\sin\theta_c d\theta_c)$ at $\theta_c=0°$ and 90°. We obtain

$$b=-1.80/\pi \text{ and } c=1.79/\pi^2 . \qquad (6)$$

Using these values in (4) we get Oberacker's estimate of θ_c vs θ_i [short dashed line in Fig. 2(a)]. We notice that at the distance of closest approach, Oberacker predicts a much larger change in orientation angles, by as much as a factor of 2, than we obtained in our classical dynamical calculation. For example, for initial orientations $\theta_i=45°$, we have the largest change $\Delta\theta=8°$ in orientation, to be compared with the estimated value of 15° from Oberacker's calculations.

In order to test the validity of our ansatz (4) for Oberacker's calculations, we have compared in Fig. 3 his orientation probability $dP/(\sin\theta_c d\theta_c)$ (shown as dots) with that obtained from Eqs. (5) and (6) (dashed line). The two data sets are almost identical, indicating that Eqs. (4) to (6) are a good approximation to Oberacker's results.

In Fig. 3, we have also plotted our calculated orientation probability $dP/(\sin\theta_c d\theta_c)$ from Eq. (5) for the values of constants b and c determined from (4) by using the change in the orientation angles ($\Delta\theta=\theta_c-\theta_i$) predicted by the classical dynamical calculations. For calculating the constants b and c we notice in Eq. (5) that the slope $d\theta_i/d\theta_c$ around $\theta_c=90°$ is determined by the constant b alone and that c corrects for the slope around $\theta_c=0°$. In view of this observation, we first set $c=0$ in (4) and calculated b for the maximum value of $\Delta\theta=8°$ at $\theta_c=45°$ and then reused (4) to obtain c for $\Delta\theta=1°$ at $\theta_c=4°$. We obtained

$$b=-0.71/\pi \text{ and } c=0.42/\pi^2 . \qquad (7)$$

The solid line in Fig. 3 shows the orientation probability resulting from our calculation by using Eq. (7) in (5). We notice that at $R=R_{\min}$ the classical dynamical calculations also predict much larger probability for the occurrence of the belly-to-belly configuration, as compared to that for the occurrence of the nose-to-nose configuration, but the relative orders of magnitudes are much smaller. Our estimates are 0.68:0.28 compared to 0.95:0.15 of Oberacker.

By considering that the favorable configurations for the giant nuclear molecule formation must lie in the angular cones of 0°–35° and 145°–180°, integration of the orientation probability in these cones results in a total orientation probability of about 0.13 for each nucleus to be aligned favorably. This is almost 30% larger than that predicted by Oberacker.[4]

For the cross sections in sub-Coulomb transfer of neutrons, the importance of this result stems from the fact that the integrand of the transfer amplitude is well localized near the distance of closest approach.[7] Hence, different estimates of the probability of occurrence at $R_{\min}$ of the various oriented configurations in collisions between unpolarized ions will apparently give rise to different averages. The effect of such an averaging of the sub-Coulomb transfer of one neutron in $^{238}U+^{238}U$ is studied in detail and will be published elsewhere.[5]

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