5. (L-Tiling) Consider C_n, a $2^n \times 2^n$ checkerboard with the upper right square removed. An *L-tiling* of C_n is a tiling of C_n with L-shaped tiles (composed of three squares) with no overlaps and no square of the checkerboard left uncovered. Prove or disprove: for every $n \in \mathbb{Z}^+$, there is an L-tiling of C_n. See Figure 1 for pictures of C_1, C_2, and C_3, and Figure 2 for pictures of all possible L-shaped tiles.
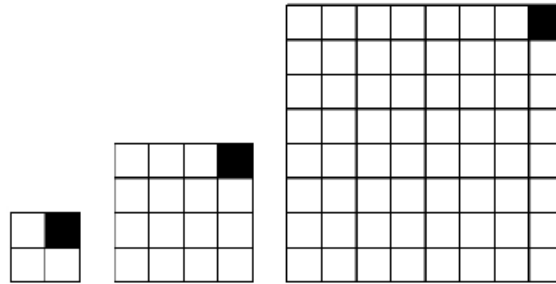

Figure 1: C_1, C_2, and C_3: the black square has been removed from the board.
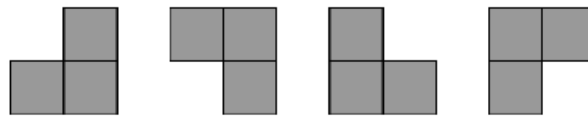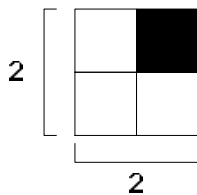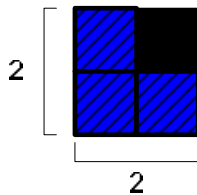

Figure 2: All possible L-tiles composed of three squares.
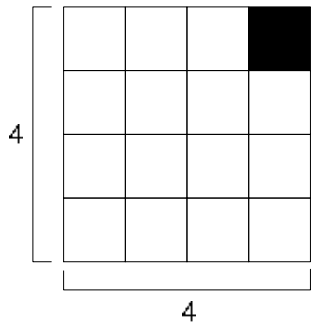
For $n = 1$, we have a 2x2 square.

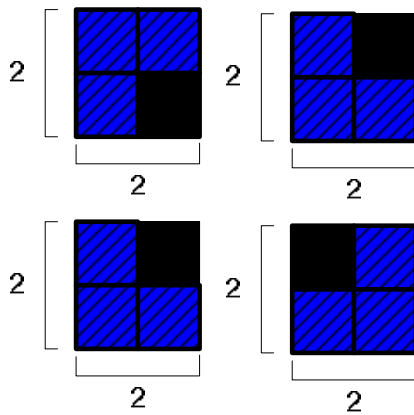We can place 1 L-Tile on the open squares.
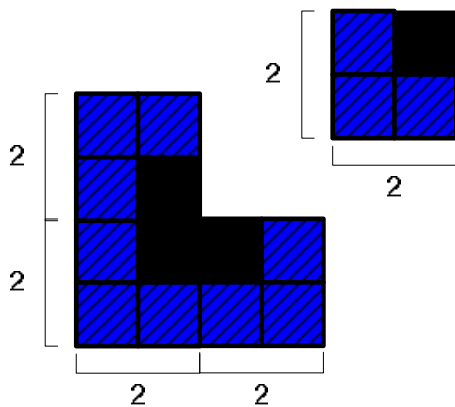

The solution is true for $n = 1$.
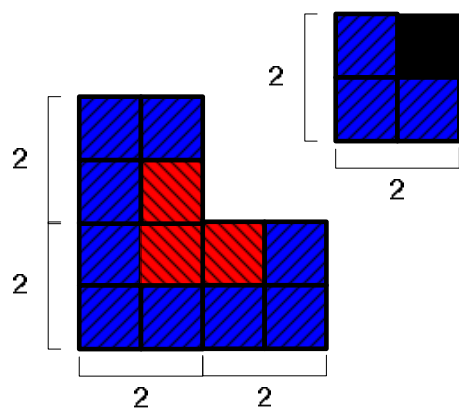
For $n = 2$, we have a 4x4 square.

We can break this square up into 4 , 2x2 squares.

We know the solution is true for a 2x2. When we combine the 2x2's we have an open area where they had a square removed.

This open area can be filled by 1 L-tile.

The solution is true for $n = 2$.

For $n = 3$, we have a 8x8 square.
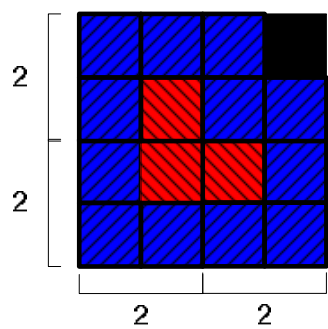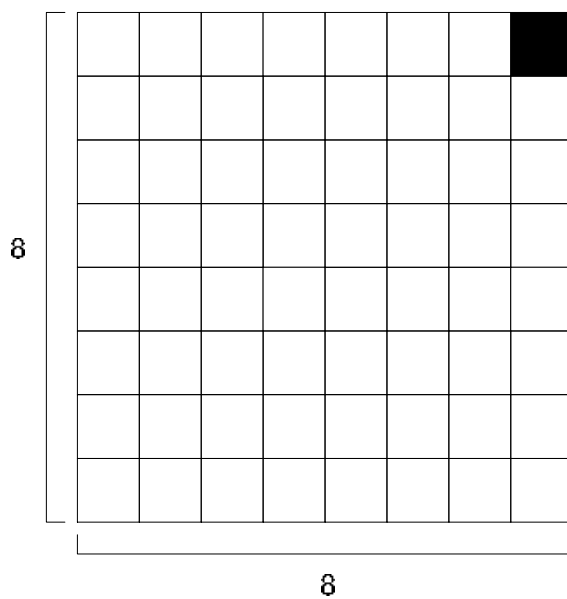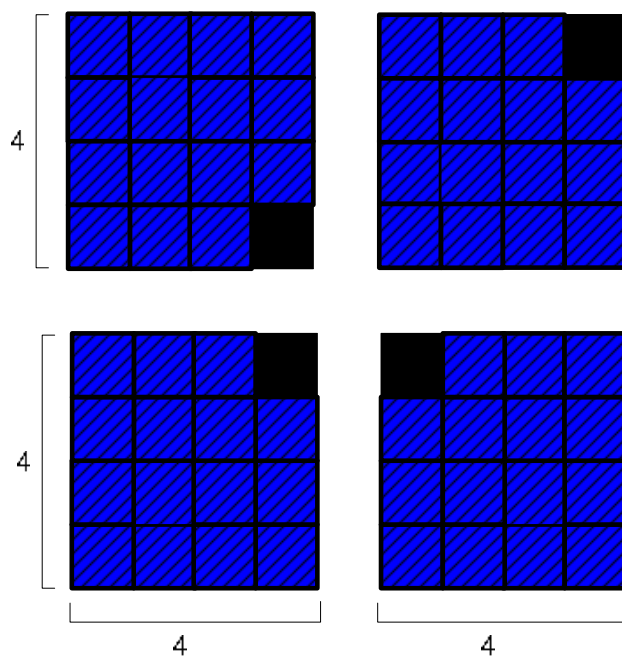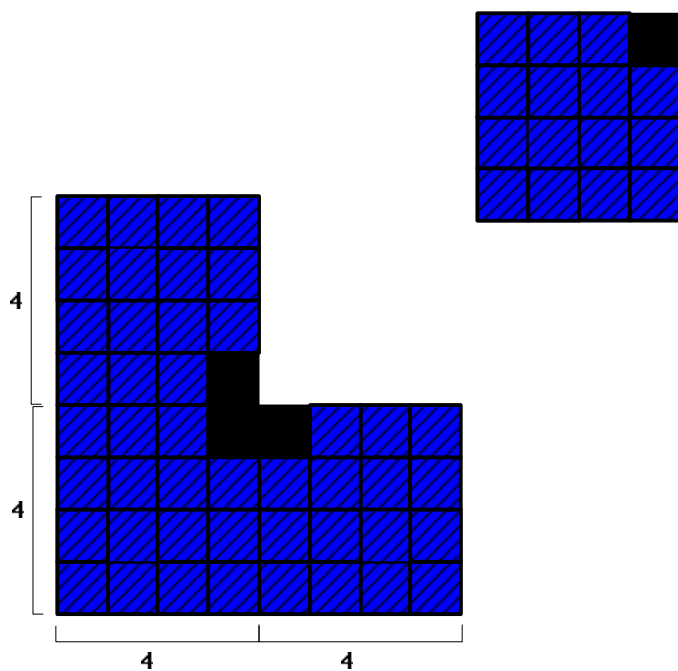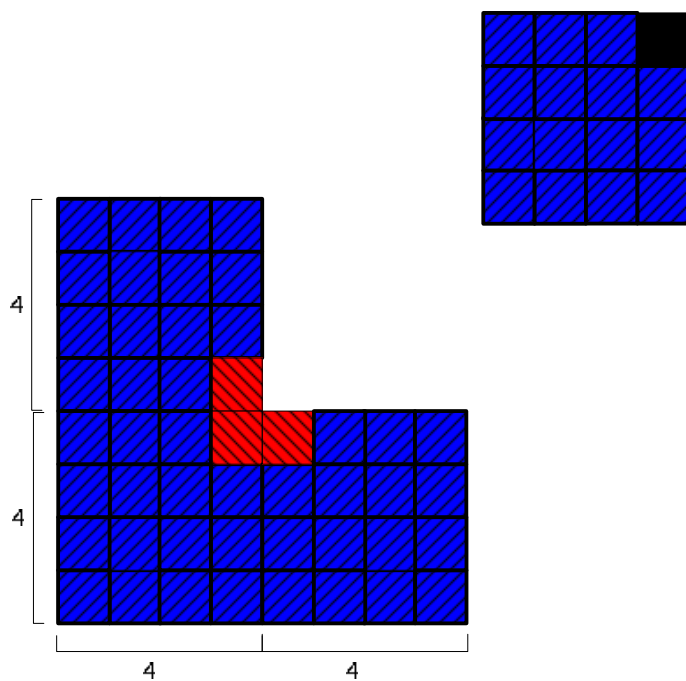
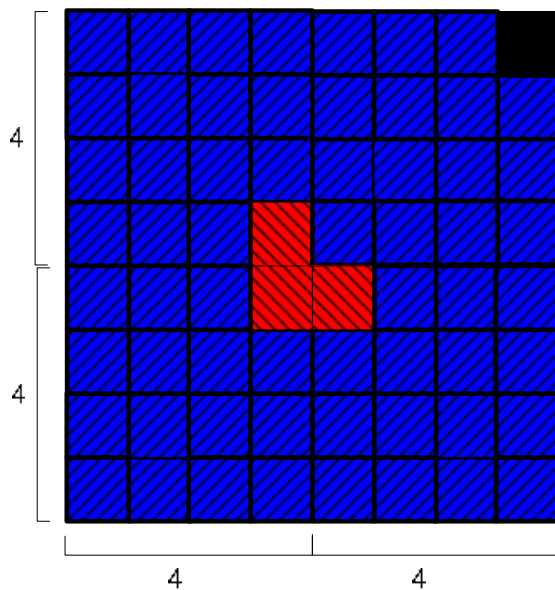

We can break this square up into 4 , 4x4 squares.

We know the solution is true for a 4x4. When we combine the 4x4's we have an open area where they had a square removed.

This open area can be filled by 1 L-tile.

The solution is true for $n = 3$.

Prove: for every $n \in \mathbb{Z}^+$, there is an L-tiling of C_n.

Need Statement, $P(n)$:

$P(n)$: a $\left(2^n \times 2^n \quad chec \ker board \right) - 1 \, square$ can be covered with L Tiles

I want to prove $P(n)$ is true for $\forall n \in \mathbb{Z}^+$.
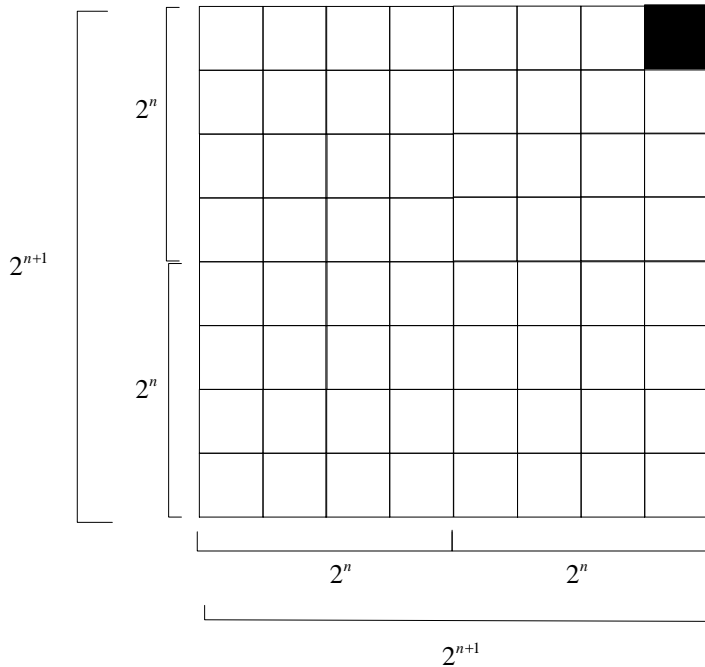
-Check some base cases:

We've already verified $P(1), P(2), P(3)$

-Induction hypothesis:

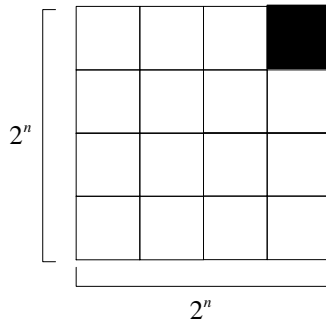There is some $n \in \mathbb{Z}^+$ such that $P(n)$ is true.

-Prove $P(n+1)$ is true.

A $2^{n+1} \times 2^{n+1}$ checkerboard with 1 square removed

NOTE: A $2^{n+1} \times 2^{n+1}$ checkerboard can be subdivided into 4, 2^n squares. These 2^n squares can be cut into 4 smaller squares, which are then cut into 4 smaller squares, and so on , and so on.

NOTE: The top right 2^n square with 1 square removed is solved by the induction hypothesis.

If we remove 1 square each from the remaining 2^n squares they are also solved by the induction hypothesis. The 3 squares removed is one L Tile.

Therefore this is true for $P(n+1)$.

-Conclusion:

There is an L Tiling of C_n for every $n \in \mathbb{Z}^+$.
Q.E.D.